11	3a	The equation of motion for a particle undergoing simple harmonic motion is $\frac{d^2x}{dt^2} = -n^2x$, where x is the displacement of the particle from the origin at time t, and n is a positive constant.	
		(i) Verify that $x = A\cos nt + B\sin nt$, where A and B are constants, is a solution of the equation of motion.	**1**
		(ii) The particle is initially at the origin and moving with velocity $2n$. Find the values of A and B in the solution $x = A\cos nt + B\sin nt$.	**2**
		(iii) When is the particle at its greatest distance from the origin?	**1**
		(iv) What is the total distance the particle travels between $t = 0$ and $t = \frac{2\pi}{n}$?	**1**

(i)
$$x = A\cos nt + B\sin nt$$
$$\frac{dx}{dt} = -An\sin nt + Bn\cos nt$$
$$\frac{d^2x}{dt^2} = -An^2\cos nt - Bn^2\sin nt$$
$$= -n^2(A\cos nt + B\sin nt)$$
$$= -n^2x$$

(ii) When $t = 0$, $x = 0$:
$$0 = A\cos 0 + B\sin 0$$
$$\therefore A = 0$$
When $t = 0$, $\frac{dx}{dt} = 2n$:
$$2n = 0 + Bn\cos 0$$
$$\therefore B = 2$$
$\therefore A = 0$ and $B = 2$

(iii) Using period $= \frac{2\pi}{n}$, the time to travel from origin to max distance is $\frac{1}{4}$ of $\frac{2\pi}{n}$

ie. $\frac{1}{4} \times \frac{2\pi}{n} = \frac{\pi}{2n}$ $\quad\therefore \frac{\pi}{2n}$ s

(iv) $\therefore x = 2\sin nt$, amplitude $= 2$
$\therefore$ particle travels 4×2 unit $= 8$ units

State Mean:
0.72/1
1.40/2
0.47/1
0.22/1

Board of Studies: Notes from the Marking Centre

(i) Some approaches made the question harder than intended. For example using the auxiliary angle method to convert the given expression into a single trigonometric expression, integrating the equation of motion twice, which required the use of later parts to establish some initial conditions to resolve the constants of integration and using $\frac{d}{dx}\left(\frac{1}{2}v^2\right)$ met with little success.

(ii) The common errors resulted from poor algebra skills in substituting $t = 0$ into nt and finding a value for n, or deducing $2n = 0 + Bn$ and concluding that $B = 0$ or $B = n$. Some candidates who were unsuccessful at, or did not attempt, part (i), differentiated the displacement correctly in order to solve (ii). However, they still did not see the significance of what they were doing in terms of part (i).

(iii) Many misinterpreted this and found where the particle was at its greatest distance rather than the required 'when'. Some candidates showed a poor understanding of the use and importance of radians by providing an answer of $t = \frac{90}{n}$.

(iv) Many candidates did not distinguish between distance and displacement and incorrectly concluded that the total distance the particle travelled was 0. Better responses recognised that $t = \frac{2\pi}{n}$ represented the end of one period, thus the particle must have travelled four times the amplitude (one complete oscillation). A common error was to conclude that it had travelled twice the amplitude.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/