11 **3b**

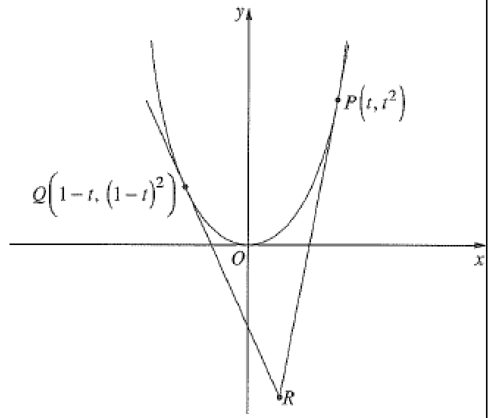
The diagram shows two distinct points $P(t, t^2)$ and $Q(1 - t, (1 - t)^2)$ on the parabola $y = x^2$. The point R is the intersection of the tangents to the parabola at P and Q.

(i) Show that the equation of the tangent to the parabola at P is $y = 2tx - t^2$. **2**

(ii) Using part (i), write down the equation of the tangent to the parabola at Q. **1**

(iii) Show that the tangents at P and Q intersect at $R(\frac{1}{2}, t - t^2)$. **2**

(iv) Describe the locus of R as t varies, stating any restriction on the y-coordinate. **2**



(i)
$$y = x^2$$
$$\frac{dy}{dx} = 2x$$
At $P(t, t^2)$: $\frac{dy}{dx} = 2t$

Using $P(t, t^2)$ and $m = 2t$:

Eqn of tangent at P: $y - y_1 = m(x - x_1)$
$$y - t^2 = 2t(x - t)$$
$$y - t^2 = 2tx - 2t^2$$
$$y = 2tx - t^2 \ \ldots (1)$$

(ii) $y - (1 - t)^2 = 2(1 - t)[x - (1 - t)]$ ..(2)

(iii) Subs $R(\frac{1}{2}, t - t^2)$ in (1) :
$$t - t^2 = 2t(\frac{1}{2}) - t^2$$
$$= t - t^2$$
$\therefore (\frac{1}{2}, t - t^2)$ lies on tangent through P

Subs $R(\frac{1}{2}, t - t^2)$ in (2) :
$$t - t^2 - (1 - t)^2 = 2(1 - t)[(\frac{1}{2}) - (1 - t]$$
$$\begin{aligned} \text{LHS} &= t - t^2 - (1 - t)^2 \\ &= t - t^2 - 1 + 2t - t^2 \\ &= 3t - 2t^2 - 1 \end{aligned}$$
$$\begin{aligned} \text{RHS} &= 2(1 - t)[(\frac{1}{2}) - (1 - t] \\ &= (2 - 2t)[-\frac{1}{2} + t] \\ &= -1 + 2t + t - 2t^2 \\ &= -1 + 3t - 2t^2 \end{aligned}$$
$\therefore$ LHS = RHS

$\therefore (\frac{1}{2}, t - t^2)$ lies on tangent through P

(iv) For R, $x = \frac{1}{2}$, $y = t - t^2$

$\therefore$ the locus of R is $x = \frac{1}{2}$.

As P and Q are distinct points, then $t \neq 1 - t$
$$2t \neq 1$$
$$t \neq \frac{1}{2}$$
$$\therefore \ y \neq \frac{1}{4}$$

State Mean:
1.88/2
0.85/1
1.44/2
0.83/2

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) Candidates should be aware that simply substituting the coordinates into the equation establishes that the point is on the line, but does not show the line is a tangent. Some responses included attempts using the memorised parametric equation $xx_1 = 2a(y + y_1)$.

(ii) Many candidates derived the equation of the tangent, rather than substituting the new parameter into the given equation from part (i). This indicates that some candidates may not understand the benefits of the parametric approach.

(iii) An appropriate approach was to substitute the coordinates of the point R into both tangents to show that it lies on both and is the point of intersection. Most candidates attempted to find the coordinates of R. Some only found the x coordinate and simply quoted the given value for y.

(iv) This part was challenging. A common response was to identify the locus as $y = t - t^2$, rather than the required $x = \frac{1}{2}$, and thus claiming that the locus was another parabola. Very few candidates were able to state the restriction on the y coordinate. Some noted the value $y = \frac{1}{4}$, although unaware of its significance.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/