11 **4a**

Consider the function $f(x) = e^{-x} - 2e^{-2x}$.

		Marks
(i)	Find $f'(x)$	**1**
(ii)	The graph $y = f(x)$ has one maximum turning point. Find the coordinates of the maximum turning point.	**2**
(iii)	Evaluate $f(\ln 2)$.	**1**
(iv)	Describe the behaviour of $f(x)$ as $x \to \infty$.	**1**
(v)	Find the y-intercept of the graph $y = f(x)$	**1**
(vi)	Sketch the graph $y = f(x)$ showing the features from parts (ii) – (v). You are not required to find any points of inflexion.	**2**

(i)
$$f(x) = e^{-x} - 2e^{-2x}$$
$$f'(x) = -e^{-x} + 4e^{-2x}$$

(ii)
$$f'(x) = -e^{-x} + 4e^{-2x} = 0$$
$$-e^{-x}(1 - 4e^{-x}) = 0$$
$$-e^{-x} \neq 0,\ e^{-x} = \frac{1}{4}$$
$$\ln e^{-x} = \ln \frac{1}{4}$$
$$-x = \ln 4^{-1}$$
$$-x = -\ln 4$$
$$x = \ln 4$$
$$f(\ln 4) = e^{-\ln 4} - 2e^{-2(\ln 4)}$$
$$= e^{\ln\frac{1}{4}} - 2e^{\ln\frac{1}{16}}$$
$$= \frac{1}{4} - \frac{1}{8}$$
$$= \frac{1}{8}$$
$$\therefore (\ln 4, \frac{1}{8})$$

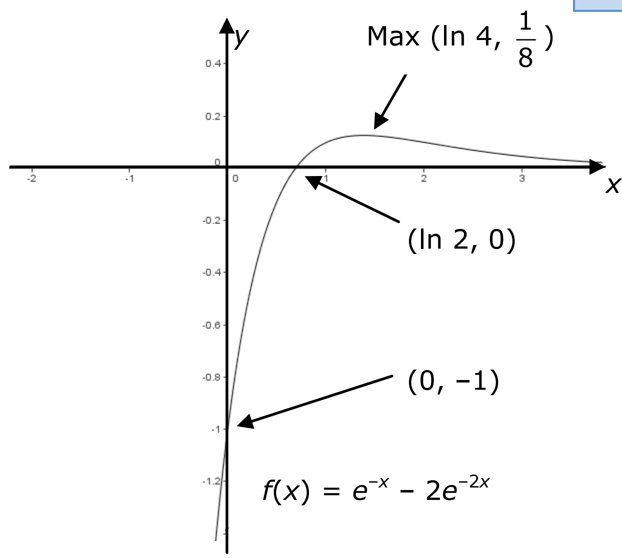
(iii)
$$f(x) = e^{-x} - 2e^{-2x}$$
$$f(\ln 2) = e^{-\ln 2} - 2e^{-2(\ln 2)}$$
$$= e^{\ln 2^{-1}} - 2e^{\ln 2^{-2}}$$
$$= \frac{1}{2} - 2 \times \frac{1}{4}$$
$$= \frac{1}{2} - \frac{1}{2}$$
$$= 0$$

(iv) As $x \to \infty$, $e^{-x} \to 0$ and $2e^{-2x} \to 0$

$\therefore e^{-x} - 2e^{-2x} \to 0$.

(v) Let $x = 0$:
$$f(x) = e^{-0} - 2e^{-2(0)}$$
$$= 1 - 2$$
$$= -1$$
$\therefore$ y-intercept of -1

(vi)



State Mean:
0.94/1
1.44/2
0.83/1
0.80/1
0.89/1
1.28/2

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

Candidates are advised to sketch large graphs (about a 1/3 of a page), clearly showing all required features. There were a number of very small graphs drawn, without clear labels.

(i) The majority of candidates were able to correctly differentiate the given function. Some candidates confused $f^{-1}(x)$ with $f'(x)$.

(ii) Some candidates, in solving the equation $f'(x)=0$, misused basic logarithmic laws. Some candidates found the x coordinate of the maximum, but could not simplify the expression involving $e^{-\log 4}-2e^{-2\log 4}$ or equivalent to find the y coordinate, or did not realise that this was required.

(iii) Some candidates entered the values into a calculator to gain the correct answer. Some did not realise that this value represented the x-intercept, even after getting the value of 0 at $x=\log 2$.

(iv) Some candidates thought the function was equal to 0 as opposed to approaching 0. This indicated a lack of understanding of the concept of a limit and the requirement to describe the behaviour. Some candidates entered very large numbers in their calculators and instead of extrapolating the limit of zero from the digits on the screen, they wrote their calculator display as the limit.

(v) This part was generally done well, although some candidates tried to find the x-intercept or left the part out, but indicated the correct intercept on the graph.

(vi) Candidates who were successful in parts (ii) to (v) were generally successful in sketching the graph, as the x-intercept, the y-intercept, the asymptote and the single maximum turning point were known.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/