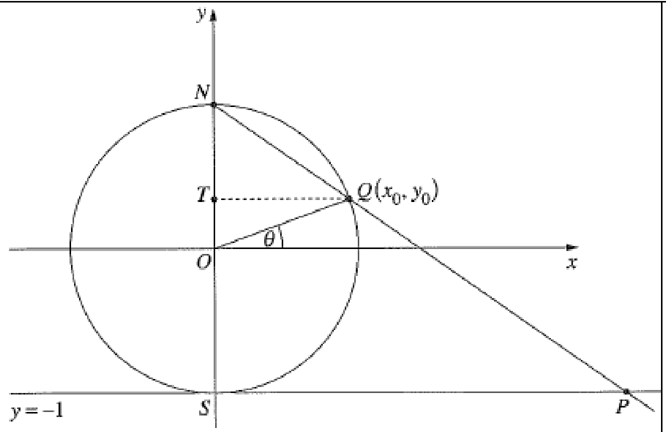
11 **5a**

In the diagram, $Q(x_0, y_0)$ is a point on the unit circle $x^2 + y^2 = 1$ at an angle θ from the positive x-axis, where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. The line through $N(0, 1)$ and Q intersects the line $y = -1$ at P. The points $T(0, y_0)$ and $S(0, -1)$ are on the y-axis.



(i) Use the fact that ΔTQN and ΔSPN are similar to show that $SP = \frac{2\cos\theta}{1-\sin\theta}$. **2**

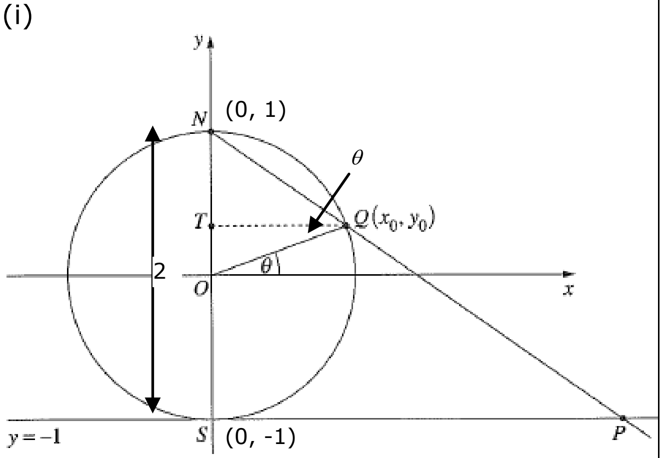
(ii) Show that $\frac{\cos\theta}{1-\sin\theta} = \sec\theta + \tan\theta$. **1**

(iii) Show that $\angle SNP = \frac{\theta}{2} + \frac{\pi}{4}$. **1**

(iv) Hence, or otherwise, show that $\sec\theta + \tan\theta = \tan\left(\frac{\theta}{2} + \frac{\pi}{4}\right)$. **1**

(v) Hence, or otherwise, solve $\sec\theta + \tan\theta = \sqrt{3}$, where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. **2**

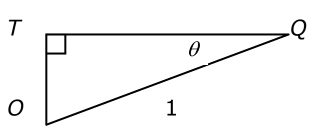
(i)



TQ || x-axis

$\therefore \angle TQO = \theta$

Also, OQ = 1



Using trig, $\frac{QT}{1} = \cos\theta$

$QT = \cos\theta$

Similarly, $OT = \sin\theta$

$\therefore NT = 1 - \sin\theta$

Using sim Δs, $\frac{NT}{NS} = \frac{QT}{SP}$

$$\frac{1-\sin\theta}{2} = \frac{\cos\theta}{SP}$$

$$\therefore SP = \frac{2\cos\theta}{1-\sin\theta}$$

(ii) RHS $= \sec\theta + \tan\theta$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{1+\sin\theta}{\cos\theta}$$

$$= \frac{1+\sin\theta}{\cos\theta} \times \frac{\cos\theta}{\cos\theta}$$

$$= \frac{\cos\theta(1+\sin\theta)}{\cos^2\theta}$$

$$= \frac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta}$$

$$= \frac{\cos\theta(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{\cos\theta}{1-\sin\theta}$$

= LHS

$$\therefore \frac{\cos\theta}{1-\sin\theta} = \sec\theta + \tan\theta.$$

State Mean:
0.81/2
0.35/1
0.11/1
0.28/1
0.90/2

(iii) $\angle QOS = \theta + \frac{\pi}{2}$ $\angle SNP = \frac{\theta}{2} + \frac{\pi}{4}$ ($\angle$ at centre is twice $\angle$ at circum. standing on same arc) (iv) Let NP intersect x-axis at X. $\therefore OX = \frac{\cos\theta}{1-\sin\theta}$ (sim Δs from (i)) $= \sec\theta + \tan\theta$ (from (ii)) Now, $\tan \angle SNP = \frac{\sec\theta + \tan\theta}{1}$ $\tan\left(\frac{\theta}{2} + \frac{\pi}{4}\right) = \frac{\sec\theta + \tan\theta}{1}$	(v) $\sec\theta + \tan\theta = \tan\left(\frac{\theta}{2} + \frac{\pi}{4}\right) = \sqrt{3}$ $\frac{\theta}{2} + \frac{\pi}{4} = \frac{\pi}{3}$ $\frac{\theta}{2} = \frac{\pi}{12}$ $\theta = \frac{\pi}{6}$

Board of Studies: Notes from the Marking Centre

(i) Candidates should be aware that when the question asks for a particular fact to be used, in this case similar triangles, then the most efficient solution is most likely to be the one that uses that fact.

(ii) This part of the question was generally done well with many different methods used. A successful approach was to multiply the left-hand side by $\frac{1+\sin\theta}{1+\sin\theta}$.

(iii)This part was poorly answered. Those few responses that used circle geometry were generally successful. The candidates who recognised the isosceles triangle and used this knowledge, usually did not fully justify their arguments. As the answer was again given, many candidates were blindly stating facts. Candidates are reminded that correct reasons must support their statements.

(iv) Many candidates successfully completed this part, despite weak setting out.

(v) This part was generally done well, with most candidates noting the relationship from part (iv).

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/