11	**5b**	To test some forensic science students, an object has been left in a park. At 10 am the temperature of the object is measured to be 30°C. The temperature in the park is a constant 22°C. The object is moved immediately to a room where the temperature is a constant 5°C. (i) The temperature of the object in the room can be modelled by the equation $T = 5 + 25e^{-kt}$, where T is the temperature of the object in degrees Celsius, t is the time in hours since the object was placed in the room and k is a constant. After one hour in the room the temperature of the object is 20°C. Show that $k = \ln\left(\frac{5}{3}\right)$. (ii) In a similar manner, the temperature of the object in the park before it was discovered can be modelled by an equation of the form $T = A + Be^{-kt}$, with the same constant $k = \ln\left(\frac{5}{3}\right)$. Find the time of day when the object had a temperature of 37°C.	**2** **3**

(i)

$$T = 5 + 25e^{-kt}$$

Let $T = 20$ and $t = 1$:

$$20 = 5 + 25e^{-k(1)}$$
$$25e^{-k} = 15$$
$$e^{-k} = \frac{3}{5}$$
$$-k = \ln\frac{3}{5}$$
$$k = \ln\frac{5}{3}$$

(ii)

$$T = A + Be^{-kt}$$

As $t \to \infty$, $T \to 22$

$$22 = A + B \times 0$$
$$A = 22$$
$$T = 22 + Be^{-kt}$$

As $t = 0$, $T = 30$

$$30 = 22 + B$$
$$B = 8$$
$$T = 22 + 8e^{-kt}$$

Let $T = 37$ and $k = \ln\left(\frac{5}{3}\right)$ ie. $-k = \ln\frac{3}{5}$

$$37 = 22 + 8 \times e^{\ln\left(\frac{3}{5}\right)t}$$
$$15 = 8 \times e^{\ln\left(\frac{3}{5}\right)t}$$
$$1.875 = e^{\ln\left(\frac{3}{5}\right)t}$$
$$\ln 1.875 = (\ln 0.6)t$$
$$t = \frac{\ln 1.875}{\ln 0.6}$$
$$= -1.23057 \ldots$$
$$= -1\text{h } 14 \text{ min}$$

It has taken 1h 14 min to drop from 37° to 30°.

$\therefore$ 10 am minus 1h 14 min = 8:46 am

State Mean: **1.82/2** **1.62/3**

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) Some candidates, after writing $k = \log\left(\frac{3}{5}\right)$, then simply stated that $k = \log\left(\frac{5}{3}\right)$, as was required by the question.

(ii) Many candidates did not correctly interpret $t = -1.23$ as a time before 10am.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/