11	6a	Use mathematical induction to prove that, for $n \ge 1$, $1 \times 5 + 2 \times 6 + 3 \times 7 + \dots + n(n+4) = \frac{1}{6}n(n+1)(2n+13)$.	3

State Mean: **2.19/3**

Step 1: Prove true for $n = 1$:

LHS $= 1 \times 5 = 5$

RHS $= \frac{1}{6} \times 1 \times (1+1)(2+13) = 5$

$\therefore$ true for $n = 1$

Step 2: Assume true for $n = k$

i.e. $S_k = 1 \times 5 + 2 \times 6 + 3 \times 7 + \dots + k(k+4) = \frac{1}{6}k(k+1)(2k+13)$.

Now prove true for $n = k + 1$

i.e. $S_k + T_{k+1} = S_{k+1}$

$$\therefore \frac{1}{6}k(k+1)(2k+13) + (k+1)(k+5) = \frac{k+1}{6}(k+1)(k+2)(2k+15)$$

$$\begin{aligned} \text{LHS} &= \frac{1}{6}k(k+1)(2k+13) + (k+1)(k+5) \\ &= \frac{k+1}{6}[k(2k+13) + 6(k+5)] \\ &= \frac{k+1}{6}[2k^2 + 13k + 6k + 30] \\ &= \frac{k+1}{6}[2k^2 + 19k + 30] \\ &= \frac{k+1}{6}(k+2)(2k+15) \\ &= \text{RHS} \end{aligned}$$

$\therefore$ true for $n = k + 1$

Step 3: As true for $n = 1$, then true for $n = 2, 3, 4, \dots$ for all integers $n \ge 1$.

Board of Studies: Notes from the Marking Centre

In most responses, candidates showed the statement true for $n = 1$ by showing their substitution. Many responses did not demonstrate an understanding that the induction process depends on assuming the statement true for $n = k$ and using this assumption to show the statement also true for $n = k + 1$. Good practice is to write the statement that is to be proved. Common errors in the proof of the statement for $n = k + 1$ included transcription errors and poor or inefficient algebraic approaches. Many candidates expanded both sides of the statement to show they were equal, often factorising a cubic into the required linear factors without any justification. The better responses involved factorisation rather than expansion. In some responses, candidates simply wrote out the structure of a mathematical induction proof without any attempt at the proof.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/