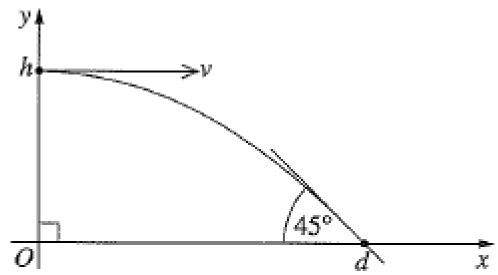
11 **6b** The diagram shows the trajectory of a ball thrown horizontally, at speed v ms^{-1}, from the top of a tower h metres above ground level. The ball strikes the ground at an angle of 45°, d metres from the base of the tower, as shown in the diagram.



The equations describing the trajectory of the ball are $x = vt$ and $y = h - \frac{1}{2}gt^2$, (Do NOT prove this.) where g is the acceleration due to gravity, and t is time in seconds.

(i) Prove that the ball strikes the ground at time $t = \sqrt{\frac{2h}{g}}$ seconds. **1**

(ii) Hence, or otherwise, show that $d = 2h$. **2**

State Mean: **0.86/1** **0.66/2**

(i) At ground level, $y = 0$

$$y = h - \frac{1}{2}gt^2$$

$$0 = h - \frac{1}{2}gt^2$$

$$\frac{1}{2}gt^2 = h$$

$$t^2 = \frac{2h}{g} \quad \text{.......... (1)}$$

$$t = \sqrt{\frac{2h}{g}}, \text{ as } t > 0$$

(ii) When ball hits ground:

Subs $x = d$ in $x = vt$,

$$d = vt$$

$$v = \frac{d}{t} \quad \text{.......... (2)}$$

Now, $\dot{x} = v$ and $\dot{y} = -gt$

As $\tan\theta = \frac{\dot{y}}{\dot{x}}$, then $\frac{-gt}{v} = \tan 45°$

$$\frac{-gt}{v} = 1$$

$$\frac{g^2t^2}{v^2} = 1 \quad \text{.......... (3)}$$

Subs (1) in (3) :

$$\frac{g^2}{v^2} \cdot \frac{2h}{g} = 1$$

$$2hg = v^2 \quad \text{.......... (4)}$$

Subs (2) in (4) :

$$2hg = \frac{d^2}{t^2}$$

$$2hg = d^2 \div \frac{2h}{g}$$

$$2hg = \frac{d^2 g}{2h}$$

$$4h^2 = d^2$$

$$2h = d$$

$$d = 2h$$

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) Most candidates correctly substituted $y = 0$ into the vertical displacement equation to show this result. Some candidates correctly verified the result by substituting $t = \sqrt{\frac{2h}{g}}$ into the vertical displacement equation to show that $y = 0$. There were some inefficient algebraic approaches with candidates first finding the cartesian equation then substituting $x = vt$ to show the result. Weaker responses stated that $y = -h$ and incorrectly obtained the result. Candidates need to be careful with notation, making sure the radical sign includes all required terms, $t = \sqrt{\frac{2h}{g}}$ not $t = \frac{\sqrt{2h}}{g}$.

(ii) In many responses, candidates stated $d = v\sqrt{\frac{2h}{g}}$ but had difficulties linking this to the velocity components and/or dealing with the negative velocity when the ball strikes the ground. Many responses were marred by fudging of negative signs when tan 45° was given as $\frac{\dot{y}}{\dot{x}}$ rather than $\frac{|\dot{y}|}{\dot{x}}$.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/