11 **6c**

A game is played by throwing darts at a target. A player can choose to throw two or three darts. Darcy plays two games. In Game 1, he chooses to throw two darts, and wins if he hits the target at least once. In Game 2, he chooses to throw three darts, and wins if he hits the target at least twice. The probability that Darcy hits the target on any throw is p, where $0 < p < 1$.

(i) Show that the probability that Darcy wins Game 1 is $2p - p^2$. **1**

(ii) Show that the probability that Darcy wins Game 2 is $3p^2 - 2p^3$. **1**

(iii) Prove that Darcy is more likely to win Game 1 than Game 2. **2**

(iv) Find the value of p for which Darcy is twice as likely to win Game 1 as he is to win Game 2. **2**

(i) Let H = hit, M = miss

$P(H) = p$, $P(M) = 1 - p$

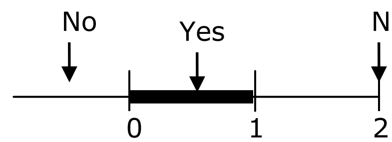
$$
\begin{aligned}
\text{P(win Game 1)} &= P(H) + P(MH) \\
&= p + (1 - p).p \\
&= p + p - p^2 \\
&= 2p - p^2
\end{aligned}
$$

(ii) P(win Game 2)

$$
\begin{aligned}
&= P(HHM) + P(MHH) + P(HMH) + P(HHH) \\
&= p^2(1 - p) + (1 - p).p^2 + p.(1 - p).p + p^3 \\
&= p^2 - p^3 + p^2 - p^3 + p^2 - p^3 + p^3 \\
&= 3p^2 - 2p^3
\end{aligned}
$$

(iii) Prove $2p - p^2 > 3p^2 - 2p^3$

$$
\begin{aligned}
3p^2 - 2p^3 - 2p + p^2 &< 0 \\
4p^2 - 2p^3 - 2p &< 0 \\
2p^3 - 4p^2 + 2p &> 0 \\
p^3 - 2p^2 + p &> 0 \\
p(p^2 - 2p + 1) &> 0 \\
p(p - 1)^2 &> 0
\end{aligned}
$$



$0 < p < 1$ … which is condition for p

$\therefore\ 2p - p^2 > 3p^2 - 2p^3$

(iv)

$$
\begin{aligned}
2p - p^2 &= 2(3p^2 - 2p^3) \\
2p - p^2 &= 6p^2 - 4p^3 \\
4p^3 - 7p^2 + 2p &= 0 \\
p(4p^2 - 7p + 2) &= 0
\end{aligned}
$$

$$p = 0, \text{ or } p = \frac{7 \pm \sqrt{49 - 4 \times 4 \times 2}}{8}$$

$$= \frac{7 \pm \sqrt{17}}{8}$$

But $0 < p < 1$, $\therefore\ p = \dfrac{7 - \sqrt{17}}{8} \approx 0.36$

State Mean:
0.47/1
0.40/1
0.34/2
0.41/2

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) Responses that made use of a tree diagram were often successful, as were those which considered $1-(1-p)^2$. In weaker responses, $\binom{2}{1}p(1-p)$ was often given as the solution. In some responses, there were insufficient or inconsistent steps in the working.

(ii) In better responses, candidates used tree diagrams or found $\binom{3}{2}p^2(1-p)+\binom{3}{3}p^3$. Some who tried to expand $1-\left[(1-p)^3+3p(1-p)^2\right]$ struggled with the algebra.

(iii) In better responses, candidates found the difference between the probabilities of winning Game 1 and Game 2 and were able to show that this difference was positive, or started with $(1-p)^2>0$ and were able to elegantly establish the inequality, sometimes reversing their steps after starting with the inequality to be proved. Some candidates attempted a proof by contradiction, but a number commenced with $2p-p^2<3p^2-2p^3$ rather than $2p-p^2\leq 3p^2-2p^3$.
Many candidates stated the inequality and proceeded until they found an expression that was positive, but sometimes were unable to justify that.
Most responses did not successfully use the domain $0<p<1$. Weaker responses showed the inequality true for a specific value of p rather than in general.

(iv) Many candidates wrote an equation that incorrectly interpreted the problem, often having Darcy being twice as likely to win Game 2 as Game 1. In many responses where candidates did correctly interpret the wording, they were unable to solve the equation or interpret their solutions over the domain.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/