12	**11f**	(i)	Use the binomial theorem to find an expression for the constant term in the expansion of $\left(2x^3 - \frac{1}{x}\right)^{12}$.	**2**
		(ii)	For what values of n does $\left(2x^3 - \frac{1}{x}\right)^{n}$ have a non-zero constant term?	**1**

(i) $\left(2x^3 - \frac{1}{x}\right)^{12}$.

For ${}^{12}C_k \,.\, (2x^3)^{12-k} \,.\, \left(\frac{-1}{x}\right)^k$,

$$\begin{aligned}
(x^3)^{12-k} \,.\, \left(\frac{-1}{x}\right)^k &= x^0 \\
x^{36-3k}.x^{-k} &= x^0 \\
36 - 3k - k &= 0 \\
36 - 4k &= 0 \\
k &= 9
\end{aligned}$$

$\therefore$ constant term is ${}^{12}C_9 \,.\, (2x^3)^3 \,.\, \left(\frac{-1}{x}\right)^9$,

or ${}^{12}C_9 \times -8 = -1760$

(ii) Consider $T_{k+1} = {}^{n}C_k \,.\, (2x^3)^{n-k} \,.\, \left(\frac{-1}{x}\right)^k$

$$\begin{aligned}
(x^3)^{n-k} \,.\, \left(\frac{-1}{x}\right)^k &= x^0 \\
x^{3n-3k}.x^{-k} &= x^0 \\
3n - 3k - k &= 0 \\
3n - 4k &= 0 \\
3n &= 4k \\
n &= \frac{4k}{3}
\end{aligned}$$

State Mean:
1.41/2
0.16/1

OR, if using

$$\begin{aligned}
(x^3)^k \,.\, \left(\frac{-1}{x}\right)^{n-k} &= x^0 \\
x^{3k}.x^{-n+k} &= x^0 \\
3k - n + k &= 0 \\
4k - n &= 0 \\
n &= 4k
\end{aligned}$$

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) In better responses, candidates clearly found the position of the constant term and then the value of the constant term. In some weaker responses, candidates did not notice the negative. Some candidates attempted to write out the whole sequence, but this approach was mostly unrewarding.

(ii) Many candidates had difficulty with interpreting the phrase 'non-zero constant' or in referring to n and k as integers. There were some rather unusual algebraic statements. Overall, candidates found this part challenging.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/