12	12a	Use mathematical induction to prove that $2^{3n} - 3^n$ is divisible by 5 for $n \geq 1$.	3

State Mean: **2.32/3**

Step 1: Prove true for $n = 1$:

$2^3 - 3^1 = 8 - 3$

$= 5$, which is divisible by 5

$\therefore$ true for $n = 1$

Step 2: Assume true for $n = k$

Let $2^{3k} - 3^k = 5M$, where M is an integer.

Now prove true for $n = k + 1$

$2^{3(k+1)} - 3^{k+1} = 2^{3k+3} - 3^{k+1}$

$= 2^{3k}.2^3 - 3^k.3$

$= 8 \times 2^{3k} - 3.3^k$

$= 5 \times 2^{3k} + 3 \times 2^{3k} - 3.3^k$

$= 5 \times 2^{3k} + 3(2^{3k} - 3^k)$

$= 5 \times 2^{3k} + 3(5M)$

$= 5[2^{3k} + 3M]$, which is divisible by 5

$\therefore$ true for $n = k + 1$

Step 3: As true for $n = 1$, then true for $n = 2, 3, 4, \ldots$ for all integers $n \geq 1$.

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

Most candidates made reasonable progress in this induction proof. Almost all showed that $P(1)$ was true and wrote statements for $P(k)$ and $P(k+1)$. However, there were some transcription errors between statements. Most candidates used their assumption to complete a proof. Candidates who demonstrated better algebraic skills established $P(k+1)$ correctly. Candidates who followed standard methods often completed efficient and correct proofs. While many candidates could write a statement like $2^{3k} - 3^k = 5M$, they did not state that M was an integer. Some candidates used unorthodox mathematical notations, such as $2^{3k} - 3^k / 5$ or $2^{3k} - 3^k = \frac{P}{5}$ which do not demonstrate clear understanding. In some weaker responses, candidates substituted twice which made the task more complicated. A handful of candidates began with the $P(k)$ statement and built it up to arrive correctly at the $P(k+1)$ statement, in an unusual but valid approach.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/