

12	12b	<p>Let $f(x) = \sqrt{4x-3}$</p> <p>(i) Find the domain of $f(x)$.</p> <p>(ii) Find an expression for the inverse function $f^{-1}(x)$.</p> <p>(iii) Find the points where the graphs $y = f(x)$ and $y = x$ intersect.</p> <p>(iv) On the same set of axes, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ showing the information found in part (iii).</p>	1 2 1 2
<p>(i) Domain: $4x - 3 \geq 0$ $4x \geq 3$ $x \geq \frac{3}{4}$</p> <p>(ii) Let $y = \sqrt{4x-3}$ For $f^{-1}(x)$: let $y \leftrightarrow x$ $x = \sqrt{4y-3}$ $x^2 = 4y - 3$ $4y = x^2 + 3$ $y = \frac{x^2 + 3}{4}$ $\therefore f^{-1}(x) = \frac{x^2 + 3}{4}$</p> <p>(iii) $\sqrt{4x-3} = x$ $4x - 3 = x^2$ $x^2 - 4x + 3 = 0$ $(x - 1)(x - 3) = 0$ $x = 1, 3$ \therefore intersect at $(1, 1), (3, 3)$</p>		<p>(iv) Graphing $y = \sqrt{4x-3}$ and $y = \frac{x^2 + 3}{4}$:</p>	

* These solutions have been provided by [projectmaths](http://projectmaths.com.au) and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

- (i) This part was well done by most candidates. In some responses, candidates did not consider that equality held, writing only an inequality. In a handful of responses, candidates confused domain with range. In some responses, candidates did not solve $4x - 3 \geq 0$ correctly.
- (ii) Almost all candidates found the correct expression for the inverse function, by the standard method of interchanging the x and y then changing the subject. In a few responses, candidates made manipulation errors, including $x = \sqrt{4y} - \sqrt{3} \Rightarrow x^2 = 4y - 3$.
- (iii) In better responses, candidates found the points of intersection or at least stated $x = 1$ or $x = 3$. Candidates who solved $\sqrt{4x-3} = x$ were more likely to obtain the correct solution than those who attempted to solve $\sqrt{4x-3} = \frac{x^2+3}{4}$. Some obtained the solution by trial and error or by plotting points.

(iv) In better responses, candidates used an accurate scale on both axes and plotted the points they had found in parts (i) and (iii). In many weaker responses, candidates did not realise that their graphs should be mirror images in the line $y = x$ and, in this case, should be semi-parabolas. In some weaker responses, candidates did not restrict their graphs to the first quadrant.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/