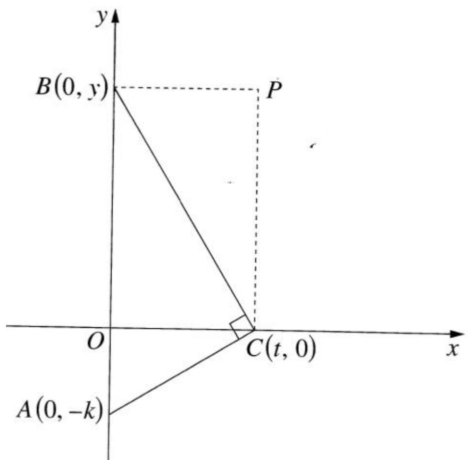
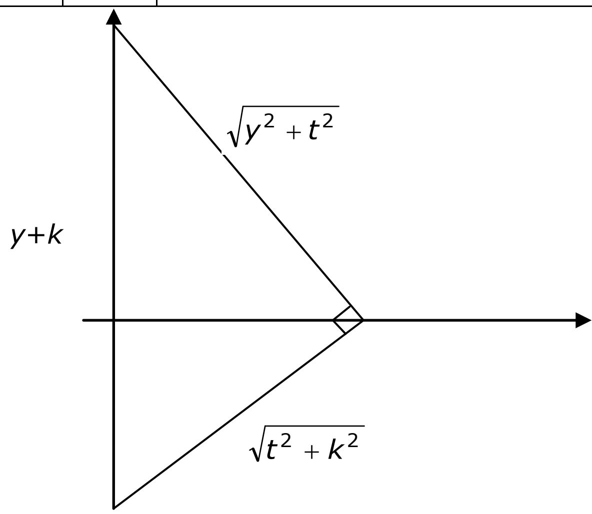
12 **12d**

Let $A(0, -k)$ be a fixed point on the y-axis with $k > 0$. The point $C(t, 0)$ is on the x-axis. The point $B(0, y)$ is on the y-axis so that ΔABC is right-angled with the right angle at C. The point P is chosen so that $OBPC$ is a rectangle as shown in the diagram.

(i) Show that P lies on the parabola given parametrically by $x = t$ and $y = \frac{t^2}{k}$ **2**

(ii) Write down the coordinates of the focus of the parabola in terms of k. **1**





(i) As P is above C, then $x = t$.

For y: using Pythagoras:

$$\left(\sqrt{y^2 + t^2}\right)^2 + \left(\sqrt{t^2 + k^2}\right)^2 = (y + k)^2$$

$$y^2 + t^2 + t^2 + k^2 = y^2 + 2ky + k^2$$

$$2t^2 = 2ky$$

$$y = \frac{t^2}{k}$$

$\therefore$ $x = t$ and $y = \frac{t^2}{k}$

State Mean: **0.60/2** **0.54/1**

OR, grad $BC = \frac{-y}{t}$, grad $AC = \frac{k}{t}$.

As $BC \perp AC$, then $\frac{-y}{t} \times \frac{k}{t} = -1$ $\therefore$ $y = \frac{t^2}{k}$

(ii) Subs $x = t$ into $y = \frac{t^2}{k}$

$$y = \frac{x^2}{k}$$

$$x^2 = ky$$

If a = focal length, then $4a = k$

$$a = \frac{k}{4}$$

$\therefore$ focus $(0, \frac{k}{4})$

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) This part was an unusual application of parameters. In better responses, candidates used the right angle given in the triangle to complete the proof successfully. The solution eluded many candidates who gave a circular proof by substituting $y = \frac{t^2}{k}$ into y. There were at least eight different approaches using the right angle, including gradients, Pythagoras' Theorem, similarity, distances, the equation of BC, areas and angles in a semicircle.

(ii) In a number of better responses, candidates deduced or stated correctly that $a = \frac{k}{4}$ and thus the focus was $\left(0, \frac{k}{4}\right)$. In weaker responses, candidates concluded that the focus was $\left(\frac{k}{4}, 0\right)$ or made other statements, suggesting a misunderstanding of the meaning of focal length.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/