12 | 13a | Write $\sin\left(2\cos^{-1}\left(\frac{2}{3}\right)\right)$ in the form $a\sqrt{b}$, where a and b are rational.

Let $\cos^{-1} \frac{2}{3} = \alpha$

State Mean: **1.04/2**

$$\therefore \cos \alpha = \frac{2}{3}$$

$$\therefore \sin \alpha = \frac{\sqrt{5}}{3}$$

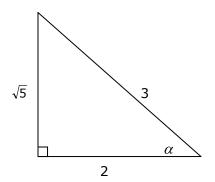
Now,
$$\sin\left(2\cos^{-1}\left(\frac{2}{3}\right)\right) = \sin 2\alpha$$

$$= 2\sin \alpha \cos \alpha$$

$$= 2 \times \frac{\sqrt{5}}{3} \times \frac{2}{3}$$

$$= \frac{4\sqrt{5}}{9}$$

$$= \frac{4}{9}\sqrt{5}$$



* These solutions have been provided by projectmaths and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

Most candidates recognised that $\cos^{-1}\frac{2}{3}$ represented an angle. In many responses,

candidates correctly used a double angle formula to find the answer. Some candidates used the *t*-results. In a significant number of weaker responses, candidates plugged the value into a calculator to produce an answer of 0.9938 ... For this part, candidates were asked to write the answer in the form $a\sqrt{b}$.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/