12	13c	A particle is moving in a straight line according to the equation $x = 5 + 6\cos 2t + 8\sin 2t$, where x is the displacement in metres and t is the time in seconds. (i) Prove that the particle is moving in simple harmonic motion by showing that x satisfies an equation of the form $\ddot{x} = -n^2(x - c)$. (ii) When is the displacement of the particle zero for the first time?	2 3

(i)
$x = 5 + 6\cos 2t + 8\sin 2t$

$\dot{x} = -12\sin 2t + 16\cos 2t$

$\ddot{x} = -24\cos 2t - 32\sin 2t$

$= -4(6\cos 2t + 8\sin 2t)$

$= -4(x - 5)$

$\therefore$ of the form $\ddot{x} = -n^2(x - c)$

(ii) Write $6\cos 2t + 8\sin 2t = R\cos(2t - \alpha)$

Now, $R = \sqrt{6^2 + 8^2} = 10$

$6\cos 2t + 8\sin 2t = 10\cos(2t - \alpha)$

$\frac{6}{10}\cos 2t + \frac{8}{10}\sin 2t = \cos(2t - \alpha)$

$\therefore \cos\alpha = \frac{6}{10}$

$\alpha = \cos^{-1}\frac{3}{5}$

$\therefore 10\cos(2t - \cos^{-1}\frac{3}{5})$

$\therefore x = 5 + 6\cos 2t + 8\sin 2t = 0$

$5 + 10\cos(2t - \cos^{-1}\frac{3}{5}) = 0$

$\cos(2t - \cos^{-1}\frac{3}{5}) = -\frac{1}{2}$

$2t - \cos^{-1}\frac{3}{5} = \frac{2\pi}{3}$

$2t = \frac{2\pi}{3} + \cos^{-1}\frac{3}{5}$

$t = 1.51$ (2 dec pl)

$\therefore$ after 1.51 seconds

State Mean:
1.62/2
1.26/3

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) Candidates were asked to verify that the given expression satisfied the equation for simple harmonic motion. This required candidates to differentiate the expression twice and then apply a simple factorisation to complete the task. Most candidates handled this. However, establishing the centre of motion ($x = 5$) did provide a challenge for some. In many weaker responses, candidates left their answer in the form $\ddot{x} = -4(6\cos 2t + 8\sin 2t)$, which is not in the form required by the question, namely $\ddot{x} = -n^2(x - c)$.

(ii) Most candidates attempted to transform the given expression into a single trigonometric expression with a minority choosing to use t-results. The centre of motion not being at the origin caused problems for a significant number of candidates who reached conclusions such as displacement of the particle is zero at the centre of motion or that the acceleration is zero when displacement is zero. Finding the first time that the displacement is zero also presented problems for some candidates. Some candidates used degrees instead of radians. A minority of candidates chose to use $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -4(x - 5)$. While this approach can lead to the correct answer, very few managed to complete the calculation.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/