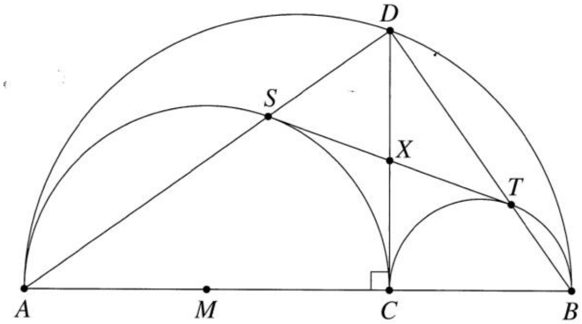
12 **14 a**

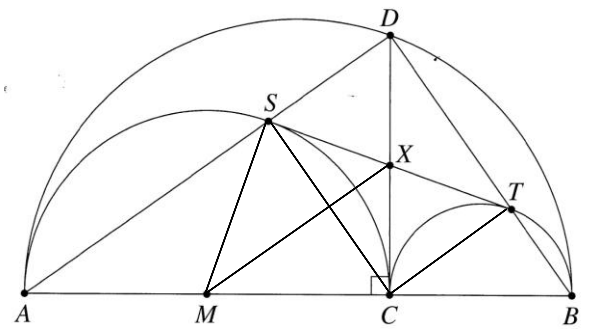
The diagram shows a large semicircle with diameter *AB* and two smaller semicircles with diameters *AC* and *BC*, respectively, where *C* is a point on the diameter *AB*. The point *M* is the centre of the semicircle with diameter *AC*. The line perpendicular to *AB* through *C* meets the largest semicircle at the point *D*. The points *S* and *T* are the intersections of the lines *AD* and *BD* with the smaller semicircles. The point *X* is the intersection of the lines *CD* and *ST*. Copy or trace the diagram into your writing booklet.



(i) Explain why *CTDS* is a rectangle. **1**

(ii) Show that ΔMXS and ΔMXC are congruent. **2**

(iii) Show that the line *ST* is a tangent to the semicircle with diameter *AC*. **1**



(i) $CS \perp AD$ (angle in semi-circle)
Similarly, $BD \perp AD$ and $BT \perp CT$
$\therefore$ *CTDS* is a rectangle.

(ii) *MX* is common
MS = *MC* (equal radii)
SX = *CX* (*X* is midpoint of diagonals of rectangle *CTDS*)
$\therefore \Delta MXS$ and ΔMXC are congruent (SSS)

(iii) $\angle$MSX = $\angle$MCX (matching $\angle$s cong Δs)
But $\angle$MCX = 90° (given)
$\therefore$ $\angle$MSX = 90°
$\therefore$ ST is tangent ($\angle$between tangent and radius is 90°)

State Mean:
0.49/1
1.21/2
0.41/1

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) Although many candidates successfully justified that *CTDS* was a rectangle, some assumed it was sufficient to show that rectangles had either a pair of equal opposite or equal adjacent angles. In a number of responses, candidates unnecessarily showed that the opposite sides were parallel or tried to reason that *CS* was greater than *CT*. In the better responses, candidates used standard terminology when presenting their geometric justifications or clearly explained the method they were attempting to use.

(ii) Although many candidates successfully completed this part, some did not use (i) to show that $SX = CX$ by using the fact that the diagonals in a rectangle are equal and bisect each other. In weaker responses, candidates often referred to 'properties of a rectangle' without specifying the properties they meant. A number of candidates assumed that *ST* was a tangent but rarely questioned this assumption when they were required to show that *ST* was a tangent in (iii). In some responses, candidates did not give supporting reasons for each step in their proof.

(iii) In many weaker responses, candidates did not provide full reasoning or see any contradictions in their reasoning with previous parts of the question, often leading to circular arguments. In some weaker responses, candidates gave inefficient reasoning about the angle in the alternate segment or equal tangents from an external point, and often did not explain why XC was a tangent first. In some weaker responses, candidates reasoned that ST was a tangent as it was perpendicular to the radius but did not provide clear reasoning. In some responses, candidates thought it sufficient to state that ST touched the semicircle once so it was a tangent to the semicircle. In better responses, candidates used the congruent triangles from (ii) to justify corresponding right angles and then correctly stated the relationship between a radius and the tangent at the point of contact.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/