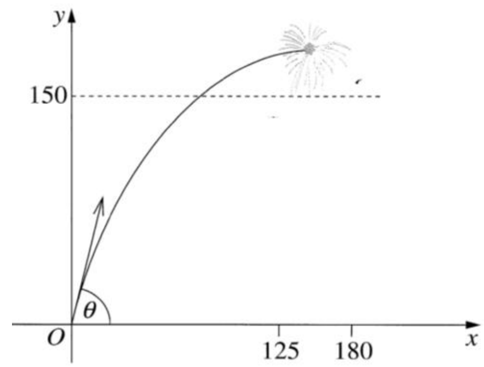
12	**14 b**	A firework is fired from O, on level ground, with velocity 70 metres per second at an inclination θ. The equations of motion of the firework are $x = 70t\cos\theta$ and $y = 70t\sin\theta - 4.9t^2$ (Do NOT prove this). The firework explodes when it reaches its maximum height.	
		(i) Show that the firework explodes at a height of $250\sin^2\theta$ metres.	**2**
		(ii) Show that the firework explodes at a horizontal distance of $250\sin 2\theta$ metres from O.	**1**
		(iii) For best viewing, the firework must explode at a horizontal distance between 125 m and 180 m from O, and at least 150 m above the ground. For what values of θ will this occur?	**3**



(i) $y = 70t\sin\theta - 4.9t^2$

$y' = 70\sin\theta - 9.8t = 0$

$$\therefore t = \frac{70\sin\theta}{9.8}$$

Subs t in y:

$$y = 70\left(\frac{70\sin\theta}{9.8}\right)\sin\theta - 4.9\left(\frac{70\sin\theta}{9.8}\right)^2$$

$= 500\sin^2\theta - 250\sin^2\theta$

$= 250\sin^2\theta$

(ii) Subs t in x:

$$x = 70\left(\frac{70\sin\theta}{9.8}\right)\cos\theta$$

$= 500\sin\theta\cos\theta$

$= 250 \times 2\sin\theta\cos\theta$

$= 250\sin 2\theta$

(iii) For best results,

$125 < x < 180$ AND $y \geq 150$:

$125 < 250\sin 2\theta < 180$

$$\frac{1}{2} < \sin 2\theta < \frac{18}{25}$$

$30^\circ < 2\theta < 46^\circ$ or $134^\circ < 2\theta < 150^\circ$

$15^\circ < \theta < 23^\circ$ or $67^\circ < \theta < 75^\circ$

Also, $250\sin^2\theta \geq 150$

$$\sin^2\theta \geq \frac{3}{5}$$

$\theta \geq 51^\circ$

$\therefore 67^\circ < \theta < 75^\circ$

State Mean:
1.25/2
0.60/1
0.83/3

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) In many responses, candidates recognised that maximum height occurred when $y' = 0$. They worked out a value for t and substituted the value for t into the parametric equation for y to find the maximum height. In a number of responses, candidates unnecessarily found the second derivative and justified that their t value was a maximum. In some weaker responses, candidates incorrectly differentiated the vertical displacement as $70\cos\theta - 9.8t$. A large number of candidates found the Cartesian equation first. These candidates often found the maximum height after inadvertently answering part (ii) with some not noticing this and reworking their solution in part (ii), sometimes with different reasoning. In some responses, candidates incorrectly differentiated the Cartesian equation by confusing variables and constants. Candidates who used the Cartesian approach were generally less successful, as were candidates who first found where the projectile hit the ground. In a smaller number of responses, candidates simply quoted a formula for the maximum height and did not show the result. Others merely applied physics formulae. A number of candidates derived the equations of motion despite having been given these. In some weaker responses, candidates took a rote approach to problem solving. A small number of attempts also included inappropriate approximations for g or t.

(ii) Most candidates who correctly answered part (i) completed this part easily. In some weaker responses, candidates simply quoted a formula for the maximum range, and did not satisfactorily establish their result.

(iii) Few candidates gained full marks on this part but many made some progress by attempting to solve the inequalities $125 < 250\sin 2\theta < 180$ and $250\sin^2\theta \geq 150$. In better responses, candidates found possible values of θ which satisfied one of the conditions for best viewing. In very few responses, candidates found the range of values that satisfied both inequalities. Generally speaking, those who solved the inequality $250\sin^2\theta \geq 150$ directly by taking the square root of both sides were much more successful than those who changed $\sin^2\theta$ to $\frac{1}{2}(1-\cos 2\theta)$. A number of responses were marred by solving equations rather than inequalities. In a large number of weaker responses, candidates unsuccessfully tried to solve this problem using a Cartesian approach and after pages of working still had not understood that the restrictions were on the maximum height and the corresponding horizontal distance. Candidates are reminded to look for links between question parts. Other candidates attempted to solve $125 < 70t\cos\theta < 180$ and $70t\sin\theta - 4.9t^2 \geq 150$, again ignoring the results of parts (i) and (ii). In many responses, candidates did not consider the domain of θ and/or 2θ and it was exceedingly rare for any candidate to consider graphs of $250\sin 2\theta$ or $250\sin^2\theta$. In weaker responses, candidates often reversed inequality signs or wrote an angle of 0.40 radians as 0.40π radians.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/