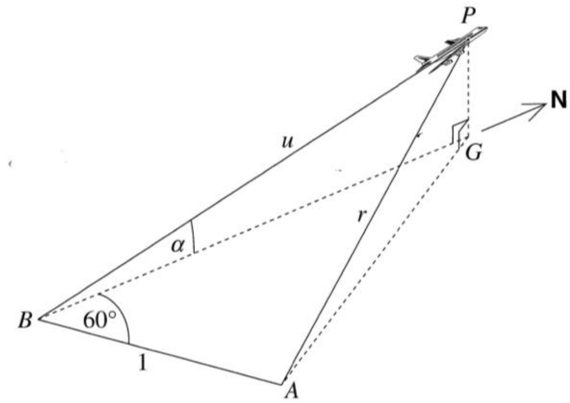
12 | **14 c**

A plane P takes off from a point B. It flies due north at a constant angle α to the horizontal. An observer is located at A, 1 km from B, at a bearing 060° from B. Let u km be the distance from B to the plane and let r km be the distance from the observer to the plane. The point G is on the ground directly below the plane.



(i) Show that $r = \sqrt{1 + u^2 - u\cos\alpha}$. **3**

(ii) The plane is travelling at a constant speed of 360 km/h.
At what rate, in terms of α, is the distance of the plane from the observer changing 5 minutes after take-off? **2**

(i) In $\triangle PBG$: $\dfrac{BG}{u} = \cos\alpha$

$BG = u\cos\alpha$

$PG = u\sin\alpha$

In $\triangle BGA$:

$GA^2 = 1^2 + u^2\cos^2\alpha - 2 \times 1 \times u\cos\alpha \times \cos 60°$

$= 1 + u^2\cos^2\alpha - u\cos\alpha$

In $\triangle PGA$:

$r^2 = PG^2 + GA^2$

$= u^2\sin^2\alpha + 1 + u^2\cos^2\alpha - u\cos\alpha$

$= 1 + u^2(\sin^2\alpha + \cos^2\alpha) - u\cos\alpha$

$= 1 + u^2 - u\cos\alpha$

$r = \sqrt{1 + u^2 - u\cos\alpha}$

(ii) As 360 km/h = 6 km/min, then $\dfrac{du}{dt} = 6$.

Also, after 5 mins, then $u = 30$

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$r = \sqrt{1 + u^2 - u\cos\alpha}$

$= (1 + u^2 - u\cos\alpha)^{\frac{1}{2}}$

$\dfrac{dr}{du} = \dfrac{1}{2}(1 + u^2 - u\cos\alpha)^{-\frac{1}{2}}.(2u - \cos\alpha)$

$= \dfrac{2u - \cos\alpha}{2\sqrt{1 + u^2 - u\cos\alpha}}$

Subs $u = 30$:

$\dfrac{dr}{du} = \dfrac{60 - \cos\alpha}{2\sqrt{901 - 30\cos\alpha}}$

Now, $\dfrac{dr}{dt} = \dfrac{dr}{du} \times \dfrac{du}{dt}$

$= \dfrac{60 - \cos\alpha}{2\sqrt{901 - 30\cos\alpha}} \times 6$

$= \dfrac{3(60 - \cos\alpha)}{\sqrt{901 - 30\cos\alpha}}$ km/min

Board of Studies: Notes from the Marking Centre

(i) Many candidates found expressions for u and/or r by considering two appropriate triangles. In better responses, candidates included the use of the cosine rule in triangle BAG. In many weaker responses, candidates misused the cosine rule in triangle PAB. In most responses, candidates made some progress by correctly providing a relationship involving u or r. In weaker responses, candidates often interchanged the trigonometric ratios for BG and PG, and were unable to proceed further. In some weaker responses, candidates introduced extraneous variables for side names, for example, $AG = c$, without clarify the meaning of their variables either on a diagram or by a statement.

(ii) Candidates needed to apply the chain rule correctly to find the rate of change of r with respect to time. The use of a chain rule was often linked to incorrect variables such as $\frac{dr}{d\alpha}$. In many responses, candidates made some progress by finding $\frac{dr}{du}$ but a large number of candidates misinterpreted variables and constants and did not correctly differentiate. Few candidates completed this part by finding the correct value of $\frac{dr}{dt}$ as they did not find either the value of $\frac{du}{dt}$, required by the application of the chain rule, or the correct value of u with consistent units being applied. Many candidates found a correct expression for $\frac{dr}{dt}$ but omitted to substitute for u. In better responses, candidates used the expression given in the question rather than an expression obtained incorrectly in part (i).

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/