13	**11 d**	Consider the function $f(x) = \dfrac{x}{4 - x^2}$. (i) Show that $f'(x) > 0$ for all x in the domain of $f(x)$. (ii) Sketch the graph of $y = f(x)$, showing all asymptotes.	**2** **2**

(i) $$f(x) = \frac{x}{4 - x^2}$$

$$f'(x) = \frac{(4 - x^2).1 - x(-2x)}{(4 - x^2)^2}$$

$$= \frac{4 - x^2 + 2x^2}{(4 - x^2)^2}$$

$$= \frac{4 + x^2}{(4 - x^2)^2}$$

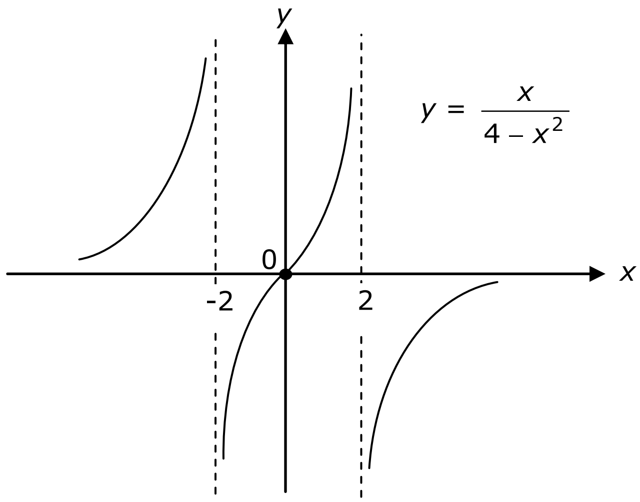
As $4 + x^2 > 0$ and $(4 - x^2)^2 > 0$,
then $f'(x) > 0$ for all x.

(ii)

- Intercepts:
  Let $x = 0$, $\therefore$ $y = 0$
- From (i), monotonic increasing
- Continuity:
  Consider $4 - x^2 = 0$
  $x = -2, 2$
  $\therefore$ discontinuous at $x = -2, 2$
- Odd function as $f(-x) = f(x)$
- Limit:

$$\lim_{x \to \infty} \frac{x}{4 - x^2} = \lim_{x \to \infty} \frac{\frac{1}{x}}{\frac{4}{x^2} - 1}$$

$$= 0$$



State Mean:
1.30/2
1.40/2

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) This part required candidates to articulate a mathematical argument. A number of candidates were able to gain full marks in this part by finding the correct derivative in simplified form, $f'(x) = \dfrac{4 + x^2}{\left(4 - x^2\right)^2}$, and then correctly reasoning why it was positive, making reference to the numerator or denominator.

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Common problems were:

- transcription or algebraic errors in differentiation, leading to incorrect expressions such as $f'(x)=\dfrac{4-3x^2}{(4-x)^2}$ or $f'(x)=\dfrac{4+2x^2}{(4-x^2)}$ which were not possible to justify as being positive
- using the derivative in expanded form, such as $f'(x)=\dfrac{4-x^2+2x^2}{16-8x^2+x^4}$, to justify as positive, which led to many spurious arguments.

(ii) Many candidates did not link this part with part (i). Most candidates were able to sketch the two vertical asymptotes correctly labelled, or with an indication of scale.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/

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