13	11f	Use the substitution $u = e^{3x}$ to evaluate $\int_0^{\frac{1}{3}} \frac{e^{3x}}{e^{6x}+1}\,dx$.	3

$$u = e^{3x}$$
$$\frac{du}{dx} = 3e^{3x}$$
$$\frac{dx}{du} = \frac{1}{3e^{3x}}$$
$$dx = \frac{du}{3e^{3x}}$$
$$= \frac{du}{3u}$$

When $x = \frac{1}{3}$, $u = e$.

When $x = 0$, $u = 1$.

$$\int_0^{\frac{1}{3}} \frac{e^{3x}}{e^{6x}+1}\,dx = \int_{u=1}^{u=e} \frac{u}{u^2+1}\,\frac{du}{3u}$$
$$= \frac{1}{3}\int_1^e \frac{1}{1+u^2}\,du$$
$$= \frac{1}{3}\left[\tan^{-1} u\right]_1^e$$
$$= \frac{1}{3}[\tan^{-1} e - \tan^{-1} 1]$$
$$= \frac{1}{3}[\tan^{-1} e - \frac{\pi}{4}]$$

State Mean:
2.00/3

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

Candidates gained some marks in this part by correctly finding $\frac{du}{dx} = 3e^{3x}$ or changing the limits. Having done this, some candidates then used substitution successfully to obtain $\frac{1}{3}\int_1^e \frac{1}{u^2+1}du$, which then led to the inverse tan function. Of these candidates, many were able to execute the final step $\frac{1}{3}\left[\tan^{-1} e - \frac{\pi}{4}\right]$ or 0.14 to gain full marks.

Common problems were:

- reaching the step $\frac{1}{3}\left[\tan^{-1} e - \tan^{-1} 1\right]$ then writing = 8.2, indicating that they were calculating the answer in degrees rather than radians
- not being able to obtain $\frac{1}{3}\int_1^e \frac{1}{u^2+1}du$ and so integrated using logs or single terms in e^{nx}.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/