

<b>13</b>	<b>12</b> <b>c</b>	<p>A cup of coffee with an initial temperature of <math>80^{\circ}\text{C}</math> is placed in a room with a constant temperature of <math>22^{\circ}\text{C}</math>. The temperature, <math>T^{\circ}\text{C}</math>, of the coffee after <math>t</math> minutes is given by <math>T = A + Be^{-kt}</math>, where <math>A</math>, <math>B</math> and <math>k</math> are positive constants. The temperature of the coffee drops to <math>60^{\circ}\text{C}</math> after 10 minutes. How long does it take for the temperature of the coffee to drop to <math>40^{\circ}\text{C}</math>? Give your answer to the nearest minute.</p>	<b>3</b>
<p>Subs <math>t = \infty</math> and <math>T = 22</math> in <math>T = A + Be^{-kt}</math></p> $22 = A + 0$ $A = 22$ <p>Subs <math>t = 0</math> and <math>T = 80</math> in <math>T = 22 + Be^{-kt}</math></p> $80 = 22 + Be^0$ $B = 80 - 22$ $= 58$ $\therefore T = 22 + 58e^{-kt}$ <p>Subs <math>t = 10</math> and <math>T = 60</math> in <math>T = 22 + 58e^{-kt}</math></p> $60 = 22 + 58e^{-k(10)}$ $58e^{-10k} = 38$ $-10k = \frac{38}{58}$ $k = \frac{\log_e \frac{38}{58}}{-10}$ $= 0.042285685 \dots$		<p>Subs <math>T = 40</math> in <math>T = 22 + 58e^{-kt}</math>:</p> $40 = 22 + 58e^{-kt}$ $58e^{-kt} = 18$ $e^{-kt} = \frac{18}{58}$ $-kt = \log_e \frac{18}{58}$ $t = \frac{\log_e \frac{18}{58}}{-k}$ $= 27.67062306 \dots$ $= 28 \text{ (nearest whole)}$ <p><math>\therefore</math> about 28 minutes</p>	<p>State Mean: <b>2.41/3</b></p>

\* These solutions have been provided by [projectmaths](http://projectmaths.com.au) and are not supplied or endorsed by the Board of Studies

### Board of Studies: Notes from the Marking Centre

Most candidates gained full marks for this part.

Common problems were:

- not identifying correct  $A$  and  $B$  values
- as a result of having incorrect  $A$  and  $B$  values, the subsequent log equation involved the log of a negative number, which was ignored
- truncating the  $k$ -value, leading to a less accurate answer.

Source: [http://www.boardofstudies.nsw.edu.au/hsc\\_exams/](http://www.boardofstudies.nsw.edu.au/hsc_exams/)