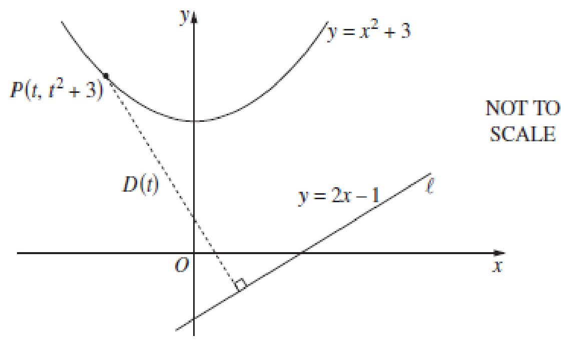
13 **12 d**

The point $P(t, t^2 + 3)$ lies on the curve $y = x^2 + 3$. The line ℓ has equation $y = 2x - 1$. The perpendicular distance from P to the line ℓ is $D(t)$.



(i) Show that $D(t) = \dfrac{t^2 - 2t + 4}{\sqrt{5}}$. **2**

(ii) Find the value of t when P is closest to ℓ. **1**

(iii) Show that, when P is closest to ℓ, the tangent to the curve at P is parallel to ℓ. **1**

(i) Using $(t, t^2 + 3)$ and $2x - y - 1 = 0$:

$$d = \left|\frac{2t - (t^2 + 3) - 1}{\sqrt{2^2 + (-1)^2}}\right|$$

$$= \left|\frac{2t - t^2 - 3 - 1}{\sqrt{4 + 1}}\right|$$

$$= \left|\frac{-t^2 - 2t - 4}{\sqrt{5}}\right|$$

$$\therefore\ D(t) = \frac{t^2 - 2t + 4}{\sqrt{5}}$$

(ii) Find t when minimum D:

$$D'(t) = \frac{2t - 2}{\sqrt{5}} = 0$$

$$t = 1$$

$$D''(t) = \frac{2}{\sqrt{5}} > 0\ \therefore\ \text{minimum}$$

$\therefore$ closest when $t = 1$

(iii) gradient of ℓ = 2

Also, for min distance, P is (1, 4).

For $y = x^2 + 3$:

$$\frac{dy}{dx} = 2x$$

$$\therefore\ \frac{dy}{dx}(1) = 2$$

$\therefore$ gradients same $\therefore$ parallel

State Mean:
0.80/2
0.64/1
0.61/1

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) Candidates were required to use the perpendicular distance formula appropriately, showing correct substitutions. It was also necessary to clearly justify/explain the removal of the absolute value sign.

(ii) Candidates needed to recognise that this was a minimisation problem, and to correctly solve $D'(t) = 0$.

(iii) Candidates needed to use $y = x^2 + 3$ to establish the gradient y' , use the substitution of $t = 1$ from (ii), and then link it to the gradient of $y = 2x - 1$. (Note: Some candidates were able to get full marks for (ii) and (iii) without necessarily getting (i) correct.)

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/