13	13 a	A spherical raindrop of radius r metres loses water through evaporation at a rate that depends on its surface area. The rate of change of the volume V of the raindrop is given by $\frac{dV}{dt} = -10^{-4}A$, where t is time in seconds and A is the surface area of the raindrop. The surface area and the volume of the raindrop are given by $A = 4\pi r^2$ and $V = \frac{4}{3}\pi r^3$ respectively. (i) Show that $\frac{dr}{dt}$ is constant. (ii) How long does it take for a raindrop of volume 10^{-6} m^3 to completely evaporate?	1 2

(i) $\frac{dV}{dt} = -10^{-4}A$

Now, $V = \frac{4}{3}\pi r^3$

$\therefore \frac{dV}{dr} = 4\pi r^2$

$= A$

Now, $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$

$-10^{-4}A = A \times \frac{dr}{dt}$

$\frac{dr}{dt} = -10^{-4}$, which is constant.

(ii) As $\frac{dr}{dt} = -10^{-4}$,

then $r = -10^{-4}t + c$ ....①

Subs $V = 10^{-6}$ in $V = \frac{4}{3}\pi r^3$

$10^{-6} = \frac{4}{3}\pi r^3$

$r^3 = 10^{-6} \div \frac{4\pi}{3}$

$r = \sqrt[3]{10^{-6} \div \frac{4\pi}{3}}$

As $\frac{dr}{dt}$ is constant, then

$t = \frac{\text{volume}}{\text{rate}}$

$= \sqrt[3]{10^{-6} \div \frac{4\pi}{3}} \div 10^{-4}$

$= 62.03504909 \ldots$

$= 62$ (nearest whole)

$\therefore$ 62 seconds

State Mean:
0.71/1
0.68/2

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) Most candidates successfully applied the chain rule and showed that the correct result was $\frac{dr}{dt} = -10^{-4}$.

Common problems were:

- not simplifying and hence the result was not a constant
- incorrect derivatives.

(ii) A common method used to find t was to integrate the rate found in (a) (i).

Most candidates tried to determine the initial radius.

Common problems were:

- errors in making r the subject of $\frac{4}{3}\pi r^3 = 10^{-6}$
- assuming that one could find the volume in terms of t from $\frac{dV}{dt} = -10^{-4}A$ without realising that A is a function of t.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/