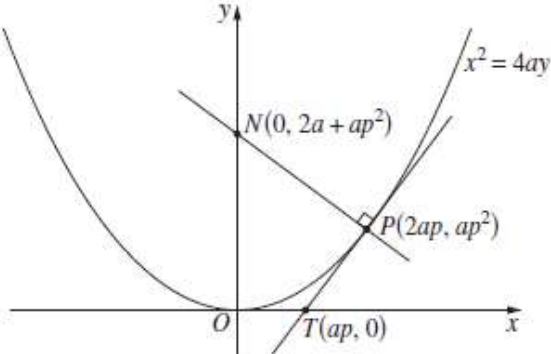


<p>13</p>	<p>13 b The point $P(2ap, ap^2)$ lies on the parabola $x^2 = 4ay$. The tangent to the parabola at P meets the x-axis at $T(ap, 0)$. The normal to the tangent at P meets the y-axis at $N(0, 2a + ap^2)$. The point G divides NT externally in the ratio 2:1.</p> <p>(i) Show that the coordinates of G are $(2ap, -2a - ap^2)$.</p> <p>(ii) Show that G lies on a parabola with the same directrix and focal length as the original parabola.</p>		<p>2</p> <p>2</p>
<p>(i) $N(0, 2a + ap^2)$ and $T(ap, 0)$; 2: -1</p> $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$ $= \left(\frac{2(ap) - 1(0)}{2-1}, \frac{2(0) - 1(2a + ap^2)}{2-1} \right)$ $= (2ap, -2a - ap^2)$	<p>(ii) $x^2 = 4ay$: focal length = a, directrix: $y = -a$</p> <p>For G: $x = 2ap \quad \therefore p = \frac{x}{2a}$</p> <p>Subs in $y = -2a - ap^2$:</p> $= -2a - a\left(\frac{x}{2a}\right)^2$ $= -2a - \frac{x^2}{4a}$ $4ay = -8a^2 - x^2$ $x^2 = -4ay - 8a^2$ $x^2 = -4a(y + 2a)$ <p>\therefore focal length = a, vertex $(0, -2a)$ and directrix: $y = -a$</p>	<p>State Mean: 1.73/2 0.61/2</p>	

* These solutions have been provided by [projectmaths](http://projectmaths.com.au) and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

- (i) Most candidates used the ratio formula correctly. Some recognised that T is the midpoint of NG and then used the midpoint formula. A few candidates used similarity or congruence to find the coordinates of G from the ratios of corresponding sides.

A common problem was:

- using $2ap + ap^2$ instead of $2a + ap^2$ for the y -coordinate of N .

- (ii) This part was challenging for most candidates.

Common problems were:

- substituting the coordinates of G into the initial parabola
- eliminating x or y and leaving the equation in terms of p
- making p the subject and then making careless errors with their algebraic manipulation

- difficulty in finding the correct equation to determine the focal length and the directrix
- stating the directrix to be $x = -a$ or $x = a$ and the focal length $4a$ units.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/