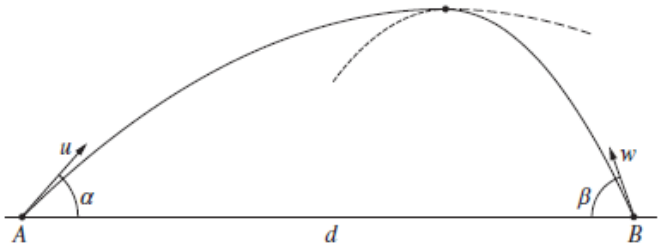
13 **13 c**

Points A and B are located d metres apart on a horizontal plane.

A projectile is fired from A towards B with initial velocity u ms^{-1} at angle α to the horizontal.



At the same time, another projectile is fired from B towards A with initial velocity w ms^{-1} and angel β to the horizontal, as shown in the diagram.
The projectiles collide when they both reach their maximum height.
The equations of motion of a projectile fired from the origin with initial velocity V ms^{-1} at angle θ to the horizontal are

$$x = Vt\cos\theta \text{ and } y = Vt\sin\theta - \frac{g}{2}t^2. \text{ (Do NOT prove this.)}$$

(i) How long does the projectile fired from A take to reach its maximum height? **2**

(ii) Show that $u\sin\alpha = w\sin\beta$ **1**

(iii) Show that $d = \frac{uw}{g}\sin(\alpha+\beta)$. **2**

(i) $y = ut\sin\alpha - \frac{g}{2}t^2$

$y' = u\sin\alpha - gt = 0$

$\therefore\ t = \frac{u\sin\alpha}{g}$ ..... (1)

(ii) Max height for projectile from B:

$t = \frac{w\sin\beta}{g}$ ..... (2)

Let (1) = (2) :

$$\frac{u\sin\alpha}{g} = \frac{w\sin\beta}{g}$$

$$u\sin\alpha = w\sin\beta$$

(iii) Subs $t = \frac{u\sin\alpha}{g}$ in $x = ut\cos\alpha$

$\therefore\ x = u\frac{u\sin\alpha}{g}\cos\alpha$

$= \frac{u^2\sin\alpha\cos\alpha}{g}$

State Mean:
1.50/2
0.58/1
0.77/2

Also, subs $t = \frac{w\sin\beta}{g}$ in $x = wt\cos\beta$

$\therefore\ x = w\frac{w\sin\beta}{g}\cos\beta$

$= \frac{w^2\sin\beta\cos\beta}{g}$

As d = sum of distance travelled by each projectile after t seconds:

$$d = \frac{u^2\sin\alpha\cos\alpha}{g} + \frac{w^2\sin\beta\cos\beta}{g}$$

From results in (ii):

$$d = \frac{uw\sin\beta\cos\alpha}{g} + \frac{uw\sin\alpha\cos\beta}{g}$$

$$= \frac{uw(\sin\beta\cos\alpha + \sin\alpha\cos\beta)}{g}$$

$$= \frac{uw\sin(\alpha+\beta)}{g}$$

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) Most candidates correctly attempted this question by solving $\dot{y} = 0$ or by finding half the time of flight.

Common problems were:

- differentiating in terms of θ instead of t
- leaving the answer in terms of v and θ instead of u and α
- ignoring the instruction 'Do NOT prove this' and deriving the given expressions for x and y.

(ii) Candidates used a variety of methods to show the result. By far the simplest approach was to equate the times taken by particles A and B to reach their maximum heights using the result found in (a) (i). Other longer solutions involved equating the values for y or $\dot{y}$ for the two particles. Some candidates used the initial vertical components of velocity to show the result, but these statements were not always accompanied by the appropriate reasoning.

(iii) Candidates found this part challenging. Most realised that the distance required was the sum of the horizontal distances travelled to reach the maximum height.

Common problems were:

- not recognising the need to substitute the results for time found in (c) (i) and (c) (ii)
- not realising the need to use the result from (c) (ii) to make the switch in the expression that leads to the final result
- finding d to average the total horizontal distance travelled by both particles and then having difficulty with the algebra involved
- those who chose to expand the given result often could not show a complete solution in reverse.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/