13	14 a			
		(i)	Show that for $k > 0$, $\dfrac{1}{(k+1)^2} - \dfrac{1}{k} + \dfrac{1}{k+1} < 0$.	1
		(ii)	Use mathematical induction to prove that for all integers $n \geq 2$, $\dfrac{1}{1^2} + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \ldots + \dfrac{1}{n^2} < 2 - \dfrac{1}{n}$.	2

State Mean:
0.25/1
0.79/2

(i)
$$\frac{1}{(k+1)^2} - \frac{1}{k} + \frac{1}{k+1} = \frac{k-(k+1)^2+k(k+1)}{k(k+1)^2}$$
$$= \frac{k-k^2-2k-1+k^2+k}{k(k+1)^2}$$
$$= \frac{-1}{k(k+1)^2} < 0, \text{ as } k(k+1)^2 > 0$$

(ii) Prove true for $n = 2$:

$$\text{LHS} = \frac{1}{1^2} + \frac{1}{2^2} = 1\frac{1}{4} \qquad \text{RHS} = 2 - \frac{1}{2} = 1\frac{1}{2}$$

LHS < RHS

$\therefore$ true for $n = 2$

Assume true for $n = k$

i.e. $\dfrac{1}{1^2} + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \ldots + \dfrac{1}{k^2} < 2 - \dfrac{1}{k}$

Now prove true for $n = k + 1$

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots + \frac{1}{k^2} + \frac{1}{(k+1)^2} < 2 - \frac{1}{k+1}$$

i.e. $2 - \dfrac{1}{k} + \dfrac{1}{(k+1)^2} < 2 - \dfrac{1}{k+1}$

$\therefore -\dfrac{1}{k} + \dfrac{1}{(k+1)^2} < -\dfrac{1}{k+1}$

But, this is already proven in (i): $\left[\dfrac{1}{(k+1)^2} - \dfrac{1}{k} + \dfrac{1}{k+1} < 0\right]$

$\therefore$ true for $n = k + 1$

$\therefore$ Statement proven using mathematical induction for $n \geq 2$.

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) Some candidates were able to combine the terms into a single term, such as $-\dfrac{1}{k(k+1)^2}$.

Some candidates simply substituted a positive number, eg $k = 1$, into the inequality and obtained a negative value. It should be noted that this method only shows that the inequality is true for that particular value, and not all positive values.

Common problems were:

- poor algebraic manipulation of the expression
- poor handling of the signs of the terms in the numerator, after creating a common denominator
- not justifying why their final expression is always negative and just assuming that 'it is obvious'.

(ii) Candidates were expected to prove the inequality using the technique of mathematical induction. Simply proving the result for the base case of $n = 2$ presented many candidates with problems.

Common errors included:

- using $n = 1$, leading to the incorrect conclusion that $1 < 1$
- using $n = 3$, possibly because 3 is the first integer greater than 2
- treating the LHS as a single term of $\frac{1}{2^2}$, instead of $\frac{1}{1^2} + \frac{1}{2^2}$ in trying to prove for $n = 2$; as it was an inequality, candidates who did this did not realise their error as $\frac{1}{2^2}$ is indeed $< 2 - \frac{1}{2}$
- not realising that in induction problems involving a series of terms, it is the sum of terms in the LHS that is being compared to the RHS, and not the general term
- not understanding the difference between proving an inequality and solving an inequation.

After making the correct assumption of $\frac{1}{1^2} + \frac{1}{2^2} + \ldots + \frac{1}{k^2} < 2 - \frac{1}{k}$, many candidates started their proof with: $\frac{1}{1^2} + \frac{1}{2^2} + \ldots + \frac{1}{k^2} + \frac{1}{(k+1)^2} = 2 - \frac{1}{k} + \frac{1}{(k+1)^2}$ or implied this by simply starting their proof with $2 - \frac{1}{k} + \frac{1}{(k+1)^2}$ instead of the correct substitution of $\frac{1}{1^2} + \frac{1}{2^2} + \ldots + \frac{1}{k^2} + \frac{1}{(k+1)^2} < 2 - \frac{1}{k} + \frac{1}{(k+1)^2}$.

Candidates who noticed the connection with part (i) were then able to use it to quickly complete the proof.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/