13	14c	The equation $e^t = \frac{1}{t}$ has an approximate solution $t_0 = 0.5$. (i) Use one application of Newton's method to show that $t_1 = 0.56$ is another approximate solution of $e^t = \frac{1}{t}$. (ii) Hence, or otherwise, find an approximation to the value of r for which the graphs $y = e^{rx}$ and $y = \log_e x$ have a common tangent at their point of intersection.	2 3

(i) Let $f(x) = e^t - \frac{1}{t}$

$= e^t - t^{-1}$ $\therefore f(0.5) = e^{0.5} - 2$

$f'(x) = e^t + t^{-2}$

$= e^t + \frac{1}{t^2}$ $\therefore f'(0.5) = e^{0.5} + 4$

Newton's Method: $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 0.5 - \frac{e^{0.5} - 2}{e^{0.5} + 4}$

$= 0.562187301 \ldots$

$= 0.56$ (2 dec pl)

(ii) Let $t = rx$:

As two curves are $y = e^{rx}$ and $y = \log_e x$

then, $y = e^t$ and $y = \log_e \frac{t}{r}$

If two curves have a common tangent then their point of contact is a double root.

As $f(x) = e^t - \log_e \frac{t}{r}$ has double root,

[Let $g(x) = \log_e \frac{t}{r}$

$g'(x) = \frac{\frac{1}{r}}{\frac{t}{r}} = \frac{1}{t}$]

then, $f'(x) = e^t - \frac{1}{t}$ has single root.

From (i), root is at $t = rx = 0.56$ $\therefore x = \frac{0.56}{r}$

Subs in $e^{rx} = \log_e x$

$e^{0.56} = \log_e \frac{0.56}{r}$

$1.7506725 = \log_e \frac{0.56}{r}$

$\frac{0.56}{r} = e^{1.7506725}$

$\frac{0.56}{r} = 5.75847395$

$r = \frac{0.56}{5.75847395}$

$= 0.97247987 \ldots$

$= 0.97$ (2 dec pl)

State Mean:
1.14/2
0.46/3

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by the Board of Studies

Board of Studies: Notes from the Marking Centre

(i) Although most candidates knew Newton's method, many could not create a valid function.

Common problems were:

- not differentiating properly
- letting the function be $f(t) = \frac{1}{t}$ or e^{t}, which in both cases simplified the problem
- using 0.56 instead of 0.5.

(ii) Most candidates realised that $e^{rx} = \log_e x$ at the point of intersection. Not many realised that the gradients also would be equal due to the common tangent.

Hence, they did not have a second equation, $re^{x} = \frac{1}{x}$, which was also required to solve the question.

A common problem was:

- finding the value of x and not going on to find the value or r, as was required.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/