2014 12a A particle is moving in simple harmonic motion about the origin, with displacement x metres. The displacement is given by $x = 2\sin 3t$, where t is time in seconds. The motion starts when $t = 0$.

(i) What is the total distance travelled by the particle when it first returns to the origin? **1**

(ii) What is the acceleration of the particle when it is first at rest? **2**

(i) Amplitude of $x = 2\sin 3t$ is 2. $\therefore$ particle has travelled 4 metres.

(ii) $v = 6\cos 3t = 0$

$3t = \frac{\pi}{2}, \ldots$ $\therefore$ particle first at rest when $3t = \frac{\pi}{2}$

$a = -18\sin 3t$

$= -18\sin\frac{\pi}{2}$

$= -18$ $\therefore$ acceleration is -18 ms^{-1}.

State Mean:
0.60
1.37

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by BOSTES.

Board of Studies: Notes from the Marking Centre

(i) This part of the question was done poorly by a large number of candidates. Many candidates drew a diagram and based their argument for the distance to be 4 metres on their diagram. A large number of candidates used basic trigonometry to determine the amplitude (2 metres) and then realised that the particle has to travel 4 metres to return to the starting point. There were a number of candidates who used the velocity–time graph to determine the distance travelled. This method required a substantial amount of work to achieve the correct solution.

Common problems were:

- thinking that the area under the displacement–time graph represented the distance travelled
- considering the amplitude of 2 as the total distance travelled
- only determining the time taken to return to the origin or the period of motion
- confusing the distance travelled with displacement from the origin.

(ii) Most candidates obtained correct expressions for velocity and acceleration in terms of t. In better responses, candidates realised that the particle was first at rest at positive amplitude and found acceleration in terms of x, using the formula: $\ddot{x} = -9x$. Candidates who used a graphical approach for (i) generally continued with this approach into this part. These candidates were then able to determine the correct acceleration. A large number of candidates who did not attempt, or were not successful in (i) were able to achieve highly in this part.

Common problems were:

- failing to realise that velocity equals zero at rest and put $t = 0$ (or $x = 0$) instead
- not solving $v = 0$ correctly for time
- not understanding the direction of the acceleration.

NB: Candidates should be careful using terms such as *deceleration* to describe negative acceleration. Also, acceleration towards the origin does not necessarily distinguish between positive and negative acceleration. All SHM has acceleration towards the centre of motion.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/2014/pdf_doc/2014-maths-ext-1.pdf