2014 12c A particle moves along a straight line with displacement x m and velocity v ms^{-1}. **3**

The acceleration of the particle is given by $\ddot{x} = 2 - e^{-\frac{x}{2}}$.
Given that $v = 4$ when $x = 0$, express v^2 in terms of x.

Using $\frac{1}{2}v^2 = \int a\ dx$

$= \int 2 - e^{-\frac{x}{2}}\ dx$

$\frac{1}{2}v^2 = 2x + 2e^{-\frac{x}{2}} + c$

$\frac{1}{2}(4)^2 = 2(0) + 2e^0 + c$

$8 = 2 + c$

$c = 6$

$\frac{1}{2}v^2 = 2x + 2e^{-\frac{x}{2}} + 6$

$v^2 = 4x + 4e^{-\frac{x}{2}} + 12$

$v^2 = 4(x + e^{-\frac{x}{2}} + 3)$

State Mean: **2.12**

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by BOSTES.

Board of Studies: Notes from the Marking Centre

Candidates generally knew that $\ddot{x} = \frac{d}{dx}(\frac{1}{2}v^2)$ or $v\frac{dv}{dx}$ and applied this correctly to attain the desired result. Some candidates approached the question by separating variables and were generally successful. Common problems were:

- not finding the correct primitive of $e^{-\frac{x}{2}}$
- not determining the value of the constant of integration correctly
- finding $\dot{x} = v = 2x + 2e^{-\frac{x}{2}} + c$ and then squaring to get v^2
- finding $\frac{1}{2}v^2 = 2x + 2e^{-\frac{x}{2}} + 6$ but then writing $v^2 = x + e^{-\frac{x}{2}} + 3$

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/2014/pdf_doc/2014-maths-ext-1.pdf