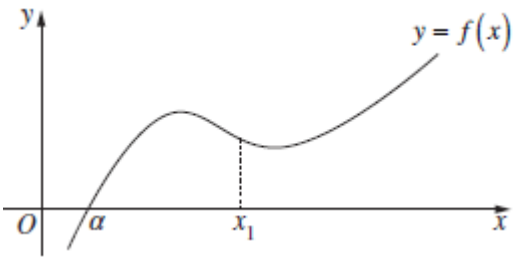
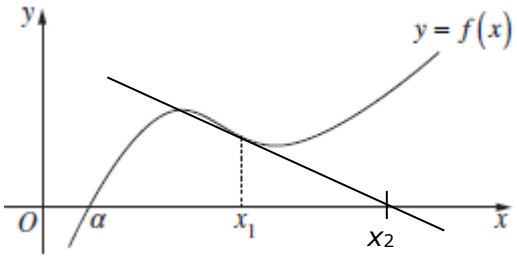
2014 12e The diagram shows the graph of a function $f(x)$. The equation $f(x) = 0$ has a root at $x = \alpha$. The value x_1, as shown in the diagram, is chosen as a first approximation of α. A second approximation, x_2, of α is obtained by applying Newton's method once, using x_1 as the first approximation. Using a diagram, or otherwise, explain why x_1 is a closer approximation than x_2.

1





The tangent to the curve at $x = x_1$ meets the x-axis at a point that is further away from $x = \alpha$ then $x = x_1$.

State Mean: **0.47**

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by BOSTES.

Board of Studies: Notes from the Marking Centre

This part was answered well by most candidates. Many candidates answered the question by drawing the diagram, showing the tangent at x_1 and an appropriate position for its x–intercept. A substantial number of candidates approached the question by referring to the formula for Newton's method and stating that because $f'(x_1) < 0$ and $f(x_1) > 0$ then $\frac{f(x_1)}{f'(x_1)} < 0$ $(-\frac{f(x_1)}{f'(x_1)} > 0)$ hence $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} > x_1$.

Candidates who argued from the formula needed to be clear with their understanding of the sign of the derivative and of the function at the first approximation for the root.

Common problems were:

- not drawing a tangent to the curve at x_1
- simply drawing x_2 on their diagram without any explanation
- basing an argument on the function values at x_1 and x_2
- failing to explain why x_1 is a better approximation – simply restating the question.

Candidates should pay attention to a question when a question states 'explain' and write clearly, re-read their statements and complete their arguments.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/2014/pdf_doc/2014-maths-ext-1.pdf