2014 **13a** Use mathematical induction to prove that $2^n + (-1)^{n+1}$ is divisible by 3 for all integers $n \geq 1$. **3**

Prove true for $n = 1$:

$2^1 + (-1)^{1+1} = 2 + 1$

$= 3$, which is divisible by 3. $\therefore$ true for $n = 1$.

Assume true for $n = k$:

Let $2^k + (-1)^{k+1} = 3M$, where M is an integer.

Now, prove true for $n = k + 1$:

$$\begin{aligned} 2^{k+1} + (-1)^{k+2} &= 2.2^k + (-1)^{k+1}.(-1) \\ &= 2.2^k - (-1)^{k+1} \\ &= 2[2^k + (-1)^{k+1}] - 3(-1)^{k+1} \\ &= 2[3M] - 3(-1)^{k+1} \\ &= 3(2M - (-1)^{k+1}) \text{, which is divisible by 3.} \end{aligned}$$

$\therefore$ true for $n = k + 1$.

$2^n + (-1)^{n+1}$ is divisible by 3 for all integers $n \geq 1$.

State Mean: **2.39**

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by BOSTES.

Board of Studies: Notes from the Marking Centre

A large number of candidates executed a correct induction proof. Those who were adept with index laws produced an efficient proof within a few lines.

Common problems were:

- knowing to use the assumption but not manipulating the resulting expression to arrive at the final step
- treating it as an equation and working on both sides.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/2014/pdf_doc/2014-maths-ext-1.pdf