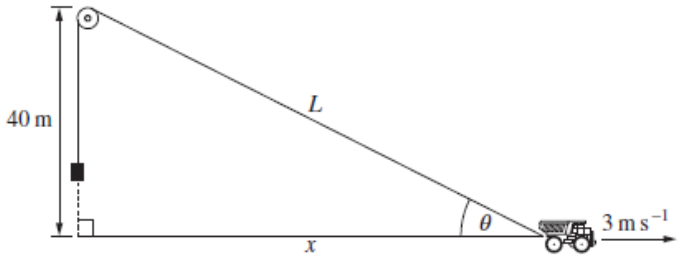
2014 13b One end of a rope is attached to a truck and the other end to a weight. The rope passes over a small wheel located at a vertical distance of 40 m above the point where the rope is attached to the truck. The distance from the truck to the small wheel is L m and the horizontal distance between them is x m. The rope makes an angle θ with the horizontal at the point where it is attached to the truck. The truck moves left to right at a constant speed of 3 ms^{-1}, as shown in the diagram.



(i) Using Pythagoras' Theorem, or otherwise, show that $\frac{dL}{dx} = \cos\theta$. **2**

(ii) Show that $\frac{dL}{dt} = 3\cos\theta$. **1**

(i)
$$L^2 = x^2 + 40^2$$
$$L = \sqrt{x^2 + 1600}$$
$$\frac{dL}{dx} = \frac{1}{2}(x^2 + 1600)^{-\frac{1}{2}}.2x$$
$$= \frac{x}{\sqrt{x^2 + 1600}}$$
$$= \frac{x}{L}$$
$$= \cos\theta$$

(ii)
$$\frac{dL}{dt} = \frac{dL}{dx} \times \frac{dx}{dt}$$
$$= \cos\theta \times 3$$
$$= 3\cos\theta$$

State Mean:
1.14
0.79

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by BOSTES.

Board of Studies: Notes from the Marking Centre

(i) Common problems were:

- being confused by the use of an angle in the diagram, many differentiated with respect to θ, leading to convoluted solutions using $\frac{dL}{d\theta}$ and $\frac{d\theta}{dx}$.
- not making the connection that $L = \sqrt{x^2 + 40^2} \Rightarrow \frac{dL}{dx} = \frac{x}{L} = \cos\theta$, leading to the incorrect substitution of $\frac{40}{\tan\theta}$ into $\frac{dL}{dx} = \frac{x}{\sqrt{x^2 + 40^2}}$ in order to eliminate x
- misunderstanding about variables as was evident in the process:

$\cos\theta = \frac{x}{L} \Rightarrow \frac{x}{\cos\theta} \Rightarrow \frac{dL}{dx} = \frac{1}{\cos\theta}$

- substituting $x = 3t \Rightarrow L = \sqrt{40^2 + (3t)^2}$ and proceeding using this.

(ii) Most candidates stated that $\frac{dx}{dt} = 3$ and arrived at the correct result.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/2014/pdf_doc/2014-maths-ext-1.pdf