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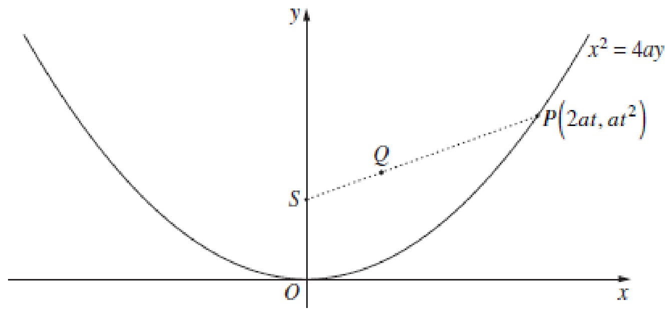
2014 **13c** The point $P(2at, at^2)$ lies on the parabola $x^2 = 4ay$ with focus S.
The point Q divides the interval PS internally in the ratio t^2:1.



(i) Show that the coordinates of Q are $x = \dfrac{2at}{1+t^2}$ and $y = \dfrac{2at^2}{1+t^2}$. **2**

(ii) Express the slope of OQ in terms of t. **1**

(iii) Using the result from part (ii), or otherwise, show that Q lies on a fixed circle of radius a. **3**

(i) $P(2at, at^2)$, $S(0, a)$, t^2:1

$$Q: \left(\frac{t^2(0)+1(2at)}{t^2+1}, \frac{t^2(a)+1(at^2)}{t^2+1}\right)$$

$$= \left(\frac{2at}{t^2+1}, \frac{2at^2}{t^2+1}\right)$$

$$\therefore x = \frac{2at}{1+t^2} \text{ and } y = \frac{2at^2}{1+t^2}$$

(ii) $$m = \frac{2at^2}{1+t^2} \div \frac{2at}{1+t^2}$$

$$= t$$

(iii) From (ii), $t = \dfrac{y}{x}$.

Now, subs in $x = \dfrac{2at}{1+t^2}$:

$$x = \frac{2a\frac{y}{x}}{1+(\frac{y}{x})^2}$$

$$x = \frac{\frac{2ay}{x}}{1+\frac{y^2}{x^2}}$$

$$x = \frac{2axy}{x^2+y^2}$$

$$x^2 + y^2 = 2ay$$

$$x^2 + y^2 - 2ay = 0$$

$$x^2 + y^2 - 2ay + a^2 = a^2$$

$$x^2 + (y - a)^2 = a^2$$

$\therefore$ locus is circle, centre $(0, a)$, radius a units.

State Mean:
1.68
0.79
0.65

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by BOSTES.

Board of Studies: Notes from the Marking Centre

(i) Those candidates who quoted the correct formula or process, substituted correctly and used basic skills such as adding like terms were able to gain full marks. Candidates who justified their answer were rewarded.

A common problem was:

- stating incorrect expressions without any basis.

(ii) This part required using the gradient formula between two points and simplifying $\dfrac{\dfrac{2at^2}{1+t^2}-0}{\dfrac{2at}{1+t^2}-0} = t$.

A common problem was:

- dealing with the compound fraction.

(iii) There were many different approaches to this part where candidates were asked to show that Q lies on a fixed circle of radius a. As the diagram was almost to scale, a large number of candidates made an intuitive guess that the centre of the circle was S since $SQ = SO$ and thus just stated the answer. Others justified it by finding the distance $SQ = a$. Although the question suggested candidates use their answer to part (c) (ii), many candidates could not see how the gradient related to a circle. Those who found and used the relation $t = \dfrac{y}{x}$ to eliminate the parameter t were often successful in reaching the result, whereas those who attempted to isolate t using $y = \dfrac{2at^2}{1+t^2}$ or $x = \dfrac{2at}{1+t^2}$ often floundered. Another approach was to create another point, say $R(0, 2a)$ and show that $m_{QR} \times m_{QO} = -1$ and then conclude that $\angle RQO$ was the angle in a semicircle. Some candidates used the gradient $t = \tan\theta$ and substituted into $y = \dfrac{2at^2}{1+t^2}$ or $x = \dfrac{2at}{1+t^2}$.

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