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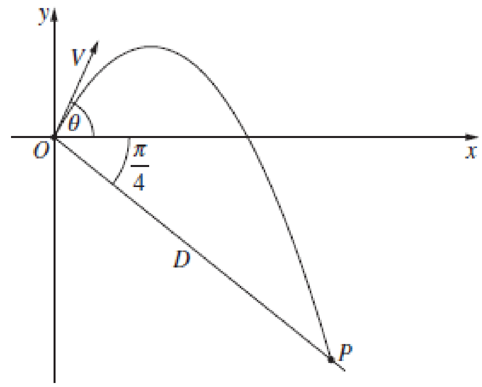
2014 **14a** The take-off point O on a ski jump is located at the top of a downslope. The angle between the downslope and the horizontal is $\frac{\pi}{4}$. A skier takes off from O with velocity V ms^{-1} at an angle θ to the horizontal, where $0 \le \theta < \frac{\pi}{2}$. The skier lands on the downslope at some point P, a distance D metres from O. The flight path of the skier is given by



$$x = Vt\cos\theta, \qquad y = -\frac{1}{2}gt^2 + Vt\sin\theta,$$

(Do NOT prove this.) where t is the time in seconds after take-off.

(i) Show that the Cartesian equation of the flight path of the skier is given by **2**

$$y = x\tan\theta - \frac{gx^2}{2V^2}\sec^2\theta.$$

(ii) Show that $D = 2\sqrt{2}\,\frac{V^2}{g}\cos\theta\,(\cos\theta + \sin\theta)$. **3**

(iii) Show that $\frac{dD}{d\theta} = 2\sqrt{2}\,\frac{V^2}{g}(\cos 2\theta - \sin 2\theta)$. **2**

(iv) Show that D has a maximum value and find the value of θ for which this occurs. **3**

(i) $x = Vt\cos\theta$

$\therefore\ t = \frac{x}{V\cos\theta}$

Subs in $y = -\frac{1}{2}gt^2 + Vt\sin\theta$:

$$y = -\frac{1}{2}g\left(\frac{x}{V\cos\theta}\right)^2 + V\left(\frac{x}{V\cos\theta}\right)\sin\theta$$

$$= x\tan\theta - \frac{gx^2}{2V^2\cos^2\theta}$$

$$\therefore\ y = x\tan\theta - \frac{gx^2}{2V^2}\sec^2\theta \quad \ldots\ldots \ (1)$$

(ii) OP has equation $y = -x$.

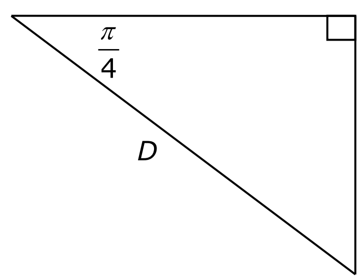
Subs in (1):

$$-x = x\tan\theta - \frac{gx^2}{2V^2}\sec^2\theta$$

$$-1 = \tan\theta - \frac{gx}{2V^2}\sec^2\theta$$

$$\frac{gx}{2V^2}\sec^2\theta = 1 + \tan\theta$$

$$\therefore\ x = \frac{2V^2}{g\sec^2\theta}(1 + \tan\theta)$$



$$\frac{x}{D} = \cos\frac{\pi}{4}$$

$$x = D\cos\frac{\pi}{4}$$

$$= \frac{D}{\sqrt{2}}$$

$$\frac{D}{\sqrt{2}} = \frac{2V^2}{g}\left(\cos^2\theta + \cos^2\theta\,\frac{\sin\theta}{\cos\theta}\right)$$

$$D = \frac{2\sqrt{2}V^2}{g}(\cos^2\theta + \cos^2\theta\,\sin\theta)$$

$$D = 2\sqrt{2}\,\frac{V^2}{g}\cos\theta\,(\cos\theta + \sin\theta)$$

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(iii)

$$D = 2\sqrt{2}\,\frac{V^2}{g}\cos\theta\,(\cos\theta + \sin\theta)$$

$$\frac{dD}{d\theta} = 2\sqrt{2}\,\frac{V^2}{g}\,[-\sin\theta\,(\cos\theta + \sin\theta) + \cos\theta\,(-\sin\theta + \cos\theta)]$$

$$= 2\sqrt{2}\,\frac{V^2}{g}\,[-\sin\theta\cos\theta - \sin^2\theta - \sin\theta\cos\theta + \cos^2\theta]$$

$$= 2\sqrt{2}\,\frac{V^2}{g}\,[\cos^2\theta - \sin^2\theta - 2\sin\theta\cos\theta]$$

$$\therefore \frac{dD}{d\theta} = 2\sqrt{2}\,\frac{V^2}{g}\,(\cos 2\theta - \sin 2\theta)$$

(iv)

$$2\sqrt{2}\,\frac{V^2}{g}\,(\cos 2\theta - \sin 2\theta) = 0$$

$$\cos 2\theta - \sin 2\theta = 0$$

$$\tan 2\theta = 1$$

$$2\theta = \frac{\pi}{4}$$

$$\theta = \frac{\pi}{8}$$

$$\frac{d^2D}{d\theta^2} = 2\sqrt{2}\,\frac{V^2}{g}\,(-2\sin 2\theta - 2\cos 2\theta)$$

$$\frac{d^2D}{d\theta^2}\left(\frac{\pi}{8}\right) = 2\sqrt{2}\,\frac{V^2}{g}\left(-2\sin\frac{\pi}{4} - 2\cos\frac{\pi}{4}\right) < 0$$

$\therefore$ D is max when $\theta = \frac{\pi}{8}$.

State Mean:
1.77
0.85
1.31
1.41

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by BOSTES.

Board of Studies: Notes from the Marking Centre

(i) The most common approach was to make t the subject in the x equation, by substitution, and eliminate it from the y equation.

(ii) This part provided the candidates with the biggest challenge in this question. There were a variety of approaches, with the most successful responses recognising that $y = -x$ and using this along with the fact that $D = 2\sqrt{x}$ to eliminate x and y from the equation in part (a)(i) and introduce D into this equation, producing something that could be rearranged into the desired result. Another common approach was to try and use Pythagoras. This method required much more algebraic manipulation and was often unsuccessful.

Common problems were:

- not dealing with the minus signs (simply changing the sign and hoping that it won't be noticed rarely works)
- attempting to apply some of the common ideas (which unfortunately played no role in the solution of this question) such as: maximum height occurs when $\dot{y} = 0$; particle hits the ground

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when y = 0, maximum range occurs when $\theta = 45^\circ$.

(iii) This was answered well by the majority of candidates. The most successful approach was to expand the parentheses in part (a)(ii) and then convert it to a double angle before differentiating.

Common problems were:

- not making a clear distinction between *D* and the derivative, and when one stops and the other begins
- putting a random collection of double angle formulae and expansion of parentheses in different positions on the page, with no clear indication of what was being used and when. (NB: A solution should follow a logical pattern which the markers can follow.)

(iv) This part was done well by many candidates. Candidates who realised that the coefficient of the derivative took no part in the process of finding the stationary point were left with the simple trig equation $\cos 2\theta - \sin 2\theta = 0$ to solve. The two main approaches to this equation were transforming the expression into a single trig function, possibly by noticing the similarity to multiple-choice Question 2, and dividing by $\cos 2\theta$ to create $\tan 2\theta = 1$. Both methods were generally successful but there were a significant number of candidates who ended up with statements such as which then produces some impractical solutions such as θ = 0 or θ = 90° or even θ = 180°.

A common problem was:

- ignoring the instruction to 'show that *D* has a maximum value'. (Candidates are reminded that showing a substitution and the value(s) when testing for maximum or minimum is essential.)

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/2014/pdf_doc/2014-maths-ext-1.pdf

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