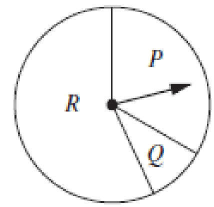
2014 14b Two players A and B play a game that consists of taking turns until a winner is determined. Each turn consists of spinning the arrow on a spinner once. The spinner has three sectors P, Q and R. The probabilities that the arrow stops in sectors P, Q and R are p, q and r respectively.



The rules of the game are as follows:

- If the arrow stops in sector P, then the player having the turn wins.
- If the arrow stops in sector Q, then the player having the turn loses and the other player wins
- If the arrow stops in sector R, then the other player takes a turn.

Player A takes the first turn.

(i) Show that the probability of player A winning on the first or second turn of the game is $(1 - r)(p + r)$. **2**

(ii) Show that the probability that player A eventually wins the game is $\dfrac{p+r}{1+r}$. **3**

(i)
$$P(A \text{ wins on first turn}) = p$$
$$P(A \text{ spins sector } R \text{ and then } B \text{ spins sector } Q) = rq$$
$$\begin{aligned} P(A \text{ wins on first or second turn}) &= p + rq \\ &= p + r(1 - p - r) \qquad \text{(as } p + q + r = 1\text{)} \\ &= p + r - pr - r^2 \\ &= p + r - r(p + r) \\ &= (1 - r)(p + r) \end{aligned}$$

(ii) $P(A \text{ wins on third turn}) = r^2p$, and $P(A \text{ wins on fourth turn}) = r^3q$.

$$\begin{aligned} P(A \text{ eventually wins}) &= p + rq + r^2p + r^3q + r^4p + r^5q + \ldots \\ &= p + rq + r^2(p + rq) + r^4(p + rq) + \ldots \\ &= (1 - r)(p + r) + r^2(1 - r)(p + r) + r^4(1 - r)(p + r) + \ldots \\ &= (1 - r)(p + r)[1 + r^2 + r^4 + \ldots] \\ &= (1 - r)(p + r)\left[\frac{1}{1 - r^2}\right] \qquad \text{(using limiting sum formula)} \\ &= (1 - r)(p + r)\frac{1}{(1 - r)(1 + r)} \\ &= \frac{p + r}{1 + r} \end{aligned}$$

State Mean:
0.50
0.34

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by BOSTES.

Board of Studies: Notes from the Marking Centre

(i) Many candidates were confused about the rules of the game and hence had trouble gaining any marks for either part of this question. A significant number of candidates simply did not attempt this question. Those who were able to interpret the rules were generally successful in showing the required result.
(ii) Only a few candidates came up with a series that would lead to a correct solution. Candidates are reminded that when using the limiting sum formula it is important to state the condition on '*r*' for the limiting sum to exist and not just assume that the series converges.

Source: http://www.boardofstudies.nsw.edu.au/hsc_exams/2014/pdf_doc/2014-maths-ext-1.pdf