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2015 11 e Use the substitution $u = 2x - 1$ to evaluate $\int_1^2 \frac{x}{(2x-1)^2} dx$. **3**

$$u = 2x - 1$$

$$\therefore x = \frac{u+1}{2}$$

$$\frac{dx}{du} = \frac{1}{2}$$

$$dx = \frac{du}{2}$$

Also, $x = 2$ then $u = 3$

$x = 1$ then $u = 1$

$$\begin{aligned} \int_1^2 \frac{x}{(2x-1)^2} dx &= \int_1^3 \frac{u+1}{2u^2} \cdot \frac{du}{2} \\ &= \int_1^3 \frac{u+1}{4u^2} du \\ &= \frac{1}{4} \left[\int_1^3 \left(\frac{1}{u} + \frac{1}{u^2} \right) du \right] \\ &= \frac{1}{4} \left[\log_e u - \frac{1}{u} \right]_1^3 \\ &= \frac{1}{4} \left[\log_e 3 - \log_e 1 - \left(\frac{1}{3} - 1 \right) \right] \\ &= \frac{1}{4} \left[\log_e 3 + \frac{2}{3} \right] \end{aligned}$$

State Mean:
2.27

* These solutions have been provided by [projectmaths](#) and are not supplied or endorsed by BOSTES.

Board of Studies: Notes from the Marking Centre

In the better responses, candidates correctly substituted to obtain the correct definite integral in terms of the variable.

Common problems were:

- incorrectly manipulating the substitution in making x the subject, often writing

$$u = 2x - 1 \text{ ® } x = \frac{u - 1}{2}$$

- not separating the integrand correctly so as to find the correct primitive
- incorrectly simplifying the definite integral, for example

$$\int_1^3 \left(\frac{u+1}{2} \right) \frac{du}{u^2} = \int_1^3 \frac{u+1}{u^2} du.$$