2015 **12 b** The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$. The equation of the chord PQ is given by $(p + q)x - 2y - 2apq = 0$. (Do NOT prove this.)

(i) Show that if PQ is a focal chord then $pq = -1$. **1**

(ii) If PQ is a focal chord and P has coordinates $(8a, 16a)$, what are the coordinates of Q in terms of a? **2**

(i) Coordinates of focus is $(0, a)$.

Substitute in $(p + q)x - 2y - 2apq = 0$:

$$(p + q)0 - 2(a) - 2apq = 0$$
$$2apq = 2a$$
$$pq = -1$$

State Mean: **0.72**

(ii)
$$2ap = 8a$$
$$p = 4$$

Substitute in $pq = -1$

$$4q = -1$$
$$q = -\frac{1}{4}$$

Coordinates of Q: $(2a(-\frac{1}{4}), a(-\frac{1}{4})^2 = Q(-\frac{a}{2}, \frac{a}{16})$

State Mean: **1.18**

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by BOSTES.

Board of Studies: Notes from the Marking Centre

(b)(i)

This part highlighted the need to understand the difference between $A \Rightarrow B$ and $B \Rightarrow A$. Candidates were given that PQ was a focal chord and asked to show that $pq = -1$. Candidates who substituted $(0, a)$ into the equation of the chord were able to derive the result.

A common problem was:

- substituting $pq = -1$ or calculating gradients with spurious justifications, suggesting that what was required was not understood.

(ii)

Candidates who recognised that $2ap = 8a \Rightarrow p = 4$ and then used $pq = -1$ to derive $q = -\frac{1}{4}$ quickly arrived at the correct coordinates $\left(\frac{-a}{2}, \frac{a}{16}\right)$.