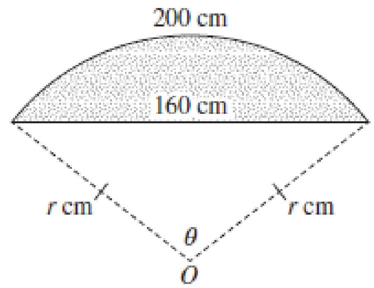
2015 **12 d**

A kitchen bench is in the shape of a segment of a circle. The segment is bounded by an arc of length 200 cm and a chord of length 160 cm. The radius of the circle is r cm and the chord subtends an angle θ at the centre O of the circle.



Not to scale

(i) Show that $160^2 = 2r^2(1 - \cos\theta)$. **1**

(ii) Hence, or otherwise, show that $8\theta^2 + 25\cos\theta - 25 = 0$. **2**

(iii) Taking $\theta_1 = \pi$ as a first approximation to the value of θ, use one application of Newton's method to find a second approximation to the value of θ. Give your answer correct to two decimal places. **2**

(i) Using the cosine rule:

$$160^2 = r^2 + r^2 - 2(r)(r)\cos\theta$$

$$160^2 = 2r^2 - 2r^2\cos\theta$$

$$160^2 = 2r^2(1 - \cos\theta)$$

State Mean: **0.77**

(ii) Now, arc length: $200 = r\theta$

$$r = \frac{200}{\theta}$$

Substitute in part (i):

$$160^2 = 2\left(\frac{200}{\theta}\right)^2(1 - \cos\theta)$$

$$25\,600 = \frac{80000}{\theta^2}(1 - \cos\theta)$$

$$8 = \frac{25}{\theta^2}(1 - \cos\theta)$$

$$8\theta^2 = 25 - 25\cos\theta$$

$$8\theta^2 + 25\cos\theta - 25 = 0$$

State Mean: **1.18**

(iii) Let $f(\theta) = 8\theta^2 + 25\cos\theta - 25$

$$f(\pi) = 8\pi^2 + 25\cos\pi - 25$$

$$= 8\pi^2 - 25 - 25$$

$$= 8\pi^2 - 50$$

$$f'(\theta) = 16\theta - 25\sin\theta$$

$$f'(\pi) = 16\pi - 25\sin\pi$$

$$= 16\pi$$

$$\theta_2 = \theta_1 - \frac{f(\theta)}{f'(\theta)}$$

$$= \pi - \frac{f(\pi)}{f'(\pi)}$$

$$= \pi - \frac{8\pi^2 - 50}{16\pi}$$

$$= 2.565514721\ldots$$

$$= 2.57 \text{ (2 dec pl)}$$

State Mean: **1.41**

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by BOSTES.

Board of Studies: Notes from the Marking Centre

(d)(i)

A large number of candidates quoted the cosine rule and substituted correctly to arrive at the required result.

A common problem was:

- not quoting the formula and/or not showing the substitutions

(ii)
Many candidates were able to execute an accurate and efficient solution by recognising that $r = \frac{200}{\theta}$, then substituting this into the equation from part (i) to eliminate r.

Common problems were:

- making algebraic slips when trying to simplify in the intermediate steps
- not relating to the information given in the question or not knowing the necessary formula $l = r\theta$

(iii)
Most candidates quoted the correct formula for Newton's method and correctly substituted their values.

A common problem was:

- not correctly differentiating $\cos\theta$