2015 **13 c** Prove by mathematical induction that for all integers $n \geq 1$, **3**

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \ldots + \frac{n}{(n+1)!} = 1 - \frac{1}{(n+1)!}.$$

Step 1: Prove true for $n = 1$:

$$\text{LHS} = \frac{1}{(1+1)!} = \frac{1}{2} \qquad \text{RHS} = 1 - \frac{1}{(1+1)!} = \frac{1}{2}$$

$\therefore$ true for $n = 1$

Step 2: Assume true for $n = k$

i.e. $S_k = \frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \ldots + \frac{k}{(k+1)!} = 1 - \frac{1}{(k+1)!}$

Now prove true for $n = k + 1$ i.e. $S_k + T_{k+1} = S_{k+1}$

$$\therefore 1 - \frac{1}{(k+1)!} + \frac{k+1}{(k+2)!} = 1 - \frac{1}{(k+2)!}$$

$$\text{LHS} = 1 - \frac{1}{(k+1)!} + \frac{k+1}{(k+2)!}$$

$$= 1 - \frac{k+2-(k+1)}{(k+2)!}$$

$$= 1 - \frac{k+2-k-1}{(k+2)!}$$

$$= 1 - \frac{1}{(k+2)!}$$

$= \text{RHS}$

$\therefore$ true for $n = k + 1$

Step 3: True for all values $n \geq 1$.

State Mean: **2.25**

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by BOSTES.

Board of Studies: Notes from the Marking Centre

Most candidates worked through the steps of an induction proof and were able to complete the proof.

Common problems were:

- not clearly setting out the proof, particularly in the inductive step
- not recognising what was the correct sum to prove
- not finding the correct denominator, and a poor understanding of factorials
- crossing out and making multiple attempts at the n = k +1 stage.