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HSC Worked Solutions

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2015 13 Prove by mathematical induction that for all integers
$$n \ge 1$$
,

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \dots + \frac{n}{(n+1)!} = 1 - \frac{1}{(n+1)!}.$$

Step 1: Prove true for n = 1:

LHS =
$$\frac{1}{(1+1)!}$$
 RHS = $1 - \frac{1}{(1+1)!}$.
= $\frac{1}{2}$ = $\frac{1}{2}$

 \therefore true for n = 1

Step 2: Assume true for n = k

i.e.
$$S_k = \frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \dots + \frac{k}{(k+1)!} = 1 - \frac{1}{(k+1)!}$$

Now prove true for n = k + 1 i.e. $S_k + T_{k+1} = S_{k+1}$

$$\therefore 1 - \frac{1}{(k+1)!} + \frac{k+1}{(k+2)!} = 1 - \frac{1}{(k+2)!}$$

$$LHS = 1 - \frac{1}{(k+1)!} + \frac{k+1}{(k+2)!}$$

$$= 1 - \frac{k+2-(k+1)}{(k+2)!}$$

$$= 1 - \frac{k+2-k-1}{(k+2)!}$$

$$= 1 - \frac{1}{(k+2)!}$$

$$= RHS$$

$$\therefore \text{ true for } n = k+1$$
Step 3: True for all values $n > 1$

Step 3: True for all values $n \ge 1$.

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Board of Studies: Notes from the Marking Centre

Most candidates worked through the steps of an induction proof and were able to complete the proof.

Common problems were:

- · not clearly setting out the proof, particularly in the inductive step
- · not recognising what was the correct sum to prove
- · not finding the correct denominator, and a poor understanding of factorials
- crossing out and making multiple attempts at the n = k +1 stage.

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State Mean: 2.25

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