2015 13 d Let $f(x) = \cos^{-1}(x) + \cos^{-1}(-x)$, where $-1 \le x \le 1$.

(i) By considering the derivative of $f(x)$, prove that $f(x)$ is constant. **2**

(ii) Hence deduce that $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$. **1**

(i) $f(x) = \cos^{-1}(x) + \cos^{-1}(-x)$

$$f'(x) = \frac{-1}{\sqrt{1-x^2}} + \frac{-1}{\sqrt{1-(-x)^2}} \times -1$$

$$= \frac{-1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}}$$

$$= 0$$

As $f'(x) = 0$, then $f(x)$ is constant.

State Mean: **1.4**

(ii) To find the constant, let $x = 0$:

$$f(0) = \cos^{-1}(0) + \cos^{-1}(-0)$$

$$= \frac{\pi}{2} + \frac{\pi}{2}$$

$$= \pi$$

As $f(x)$ is constant, then $\cos^{-1}(x) + \cos^{-1}(-x) = \pi$

$$\therefore \cos^{-1}(-x) = \pi - \cos^{-1}(x)$$

State Mean: **0.35**

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by BOSTES.

Board of Studies: Notes from the Marking Centre

(d)(i)

Candidates are encouraged to take care with writing plus and minus signs.

Common problems were:

- ignoring the question and trying to solve the problem without differentiation
- calculating the derivative incorrectly
- not understanding that a constant function has a gradient of 0.

(ii)

Candidates were generally more successful when they took the simplest approach of substituting into $f(x)$ an appropriate value (in the correct domain).

Common problems were:

- not reading the question (the word 'hence') to deduce the solution from part (i) not showing any proof
- ignoring the requirement to deduce the result by substituting into the required equation and showing the result was true.