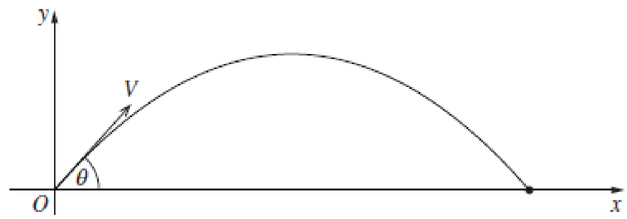
2015 14 a A projectile is fired from the origin O with initial velocity V ms^{-1} at an angle θ to the horizontal. The equations of motion are given by

$$x = Vt\cos\theta,\ y = Vt\sin\theta - \frac{1}{2}gt^2.$$

(Do NOT prove this.)



(i) Show that the horizontal range of the projectile is $\dfrac{V^2\sin 2\theta}{g}$. **2**

A particular projectile is fired so that $\theta = \dfrac{\pi}{3}$.

(ii) Find the angle that this projectile makes with the horizontal when $t = \dfrac{2V}{\sqrt{3}g}$. **2**

(iii) State whether this projectile is travelling upwards or downwards when $t = \dfrac{2V}{\sqrt{3}g}$. Justify your answer. **1**

(i) Substitute $y = 0$:

$$0 = Vt\sin\theta - \frac{1}{2}gt^2$$

$$t\left(V\sin\theta - \frac{1}{2}gt\right) = 0$$

$$t = 0,\ \frac{2V\sin\theta}{g}$$

Substitute $t = \dfrac{2V\sin\theta}{g}$:

$$x = V\left(\frac{2V\sin\theta}{g}\right)\cos\theta$$

$$= \frac{V^2\times 2\sin\theta\cos\theta}{g}$$

$$= \frac{V^2\sin 2\theta}{g}$$

State Mean: **1.54**

(ii) $\dot{x} = V\cos\theta,\ \dot{y} = V\sin\theta - gt$

Substitute $\theta = \dfrac{\pi}{3},\ t = \dfrac{2V}{\sqrt{3}g}$:

$$\dot{x} = V\cos\frac{\pi}{3}$$

$$\therefore\ \dot{x} = \frac{V}{2}$$

$$\dot{y} = V\sin\frac{\pi}{3} - g\left(\frac{2V}{\sqrt{3}g}\right)$$

$$\therefore\ \dot{y} = \frac{V\sqrt{3}}{2} - \frac{2V}{\sqrt{3}}$$

$$= \frac{3V - 4V}{2\sqrt{3}}$$

$$= \frac{-V}{2\sqrt{3}}$$

$$\text{Angle} = \tan^{-1}\left[\frac{-V}{2\sqrt{3}} \div \frac{V}{2}\right]$$

$$= \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$$

$$= -\frac{\pi}{6}$$

State Mean: **0.55**

(iii) As angle is −ve, the projectile is travelling downwards.

State Mean: **0.31**

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by BOSTES.

Board of Studies: Notes from the Marking Centre

(a)(i)
Finding the time when the particle hits the ground and then substituting this time into the horizontal displacement proved to be the most successful way of approaching this problem.

Some candidates use the symmetry of the problem, and found the time of the greatest height and then doubled this time to find the flight time.

Some candidates chose a longer solution, creating a Cartesian equation by eliminating the parameter '*t*' from the displacement equations.

(ii)
In contrast to part (i), part (ii) proved to be challenging for many candidates.

Candidates who recognised that $\tan\alpha = \frac{\dot{y}}{\dot{x}}$, were generally successful.

Common problems were:

- using the displacement equations instead of the velocity equations to find the required angle
- confusing the fixed angle of projection, with the variable angle the particle makes with the horizontal
- using the exact values for $30°$ instead of the given $\frac{\pi}{3}$.

(iii)
Generally candidates who were successful in part (ii) were successful in using it to justify their answer to part (iii).

The other successful approach was to compare the given time with the time at the greatest height, and then correctly concluding that it was after this time, so the particle must be going down.

A common problem was:

- using incorrect logic such as 'as the angle is less than the projection angle, the particle must be travelling downwards' or 'as acceleration due to gravity was negative, the particle must be travelling downwards'.