2015 **14 b** A particle is moving horizontally. Initially the particle is at the origin O moving with velocity 1 m s^{-1}. The acceleration of the particle is given by $\ddot{x} = x - 1$, where x is its displacement at time t.

(i) Show that the velocity of the particle is given by $\dot{x} = 1 - x$. **3**

(ii) Find an expression for x as a function of t. **2**

(iii) Find the limiting position of the particle. **1**

(i)

$$v^2 = 2\int x - 1 \; dx$$

$$= 2\left[\frac{x^2}{2} - x\right] + c$$

$$v^2 = x^2 - 2x + c$$

Substitute $v = 1$, $x = 0$:

$$1^2 = (0)^2 - 2(0) + c$$

$$\therefore c = 1$$

$$\therefore v^2 = x^2 - 2x + 1$$

$$= (x - 1)^2$$

$$\therefore v = \pm(x - 1)$$

But $v = 1$, $x = 0$:

$$\therefore v = -(x - 1)$$

$$\therefore \dot{x} = 1 - x$$

State Mean: **1.69**

(ii)

$$\frac{dx}{dt} = 1 - x \; dx$$

$$\therefore \frac{dt}{dx} = \int \frac{1}{1 - x} \; dx$$

$$t = -\log_e(1 - x) + k$$

Substitute $t = 0$, $x = 0$:

$$0 = -0 + k$$

$$\therefore k = 0$$

$$\therefore t = -\log_e(1 - x)$$

$$\log_e(1 - x) = -t$$

$$1 - x = e^{-t}$$

$$x = 1 - e^{-t}$$

State Mean: **0.97**

(iii) As $t \to \infty$, $e^{-t} \to 0$

$$\therefore x \to 1$$

The limiting position is $x = 1$.

State Mean: **0.42**

Board of Studies: Notes from the Marking Centre

(b)(i)

Those candidates who used $\frac{d}{dx}\left(\frac{1}{2}v^2\right)$ or $v\frac{dv}{dx}$ usually proceeded to write v^2 as a perfect square in terms of x.

Common problems were:

- not recognising that acceleration was given as a function of displacement (x) and not as a function of time (t), and simply differentiating with respect to x

- not using the initial conditions to justify why $\dot{x} = 1 - x$ (and not $\dot{x} = x - 1$), just assuming it to be so.

Responses using definite integrals were generally more successful than those using indefinite integrals.

(ii)
Common problems were:

- simply integrating the velocity equation to get a displacement equation, seemingly unconcerned with a function involving a mixture of x and t as the independent variables
- not understanding what 'x as a function of t' means, and leaving answers, with t being the subject of the formula
- not finding the constant of integration when using indefinite integrals.

(iii)
Those candidates who successfully completed part (b)(ii), found a limiting position by realising that $\lim_{t \to \infty} e^{-t} = 0$.
Another successful method was to solve $\dot{x} = 0$, which is valid for a continually increasing function.

Many candidates who left their answer in terms of x in part (b)(ii) commenced this part by successfully making x the subject.

Common problems were:

- simply giving the range of the displacement function
- not realising a limiting position must be a finite number.