2015 14 c Two players *A* and *B* play a series of games against each other to get a prize. In any game, either of the players is equally likely to win. To begin with, the first player who wins a total of 5 games gets the prize.

(i) Explain why the probability of player *A* getting the prize in exactly 7 games is $\binom{6}{4}\left(\frac{1}{2}\right)^7$. **1**

(ii) Write an expression for the probability of player *A* getting the prize in at most 7 games. **1**

(iii) Suppose now that the prize is given to the first player to win a total of $(n + 1)$ games, where n is a positive integer. **2**

By considering the probability that *A* gets the prize, prove that

$$\binom{n}{n}2^n + \binom{n+1}{n}2^{n-1} + \binom{n+2}{n}2^{n-2} + \ldots + \binom{2n}{n}2^n = 2^{2n}.$$

(i) The seventh game is won by player *A*. This means that *A* wins 4 of the first 6 games.

Let p = P(*A* not wins) = $\frac{1}{2}$ Let q = P(*A* wins) = $\frac{1}{2}$

$$\text{P}(A \text{ wins 4 of first 6}) = \binom{6}{4}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^4$$

$$\therefore \text{P}(A \text{ wins 4 of first 6 games and the 7}^{\text{th}} \text{ game}) = \binom{6}{4}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^4 \times \frac{1}{2}$$

$$= \binom{6}{4}\left(\frac{1}{2}\right)^7$$

State Mean: **0.19**

(ii) $$\text{P(prize after 5, 6 or 7 games)} = \binom{4}{4}\left(\frac{1}{2}\right)^5 + \binom{5}{4}\left(\frac{1}{2}\right)^6 + \binom{6}{4}\left(\frac{1}{2}\right)^7$$

State Mean: **0.23**

(iii) If a prize after $(n + 1)$ games, then there is a possible $2n$ number of games before a player wins the prize. Also, [as P(A wins) = P(*B* wins) = $\frac{1}{2}$]

$$\text{P}(A \text{ eventually wins the prize}) = \binom{n}{n}\left(\frac{1}{2}\right)^{n+1} + \binom{n+1}{n}\left(\frac{1}{2}\right)^{n+2} + \ldots + \binom{2n}{n}\left(\frac{1}{2}\right)^{2n+1} = \frac{1}{2}$$

Now as $2^{-1} = \frac{1}{2}$ and $2^{-1} \times 2^{2n+1} = 2^{2n}$, then multiply both sides by 2^{2n+1}:

$$\therefore \binom{n}{n}2^n + \binom{n+1}{n}2^{n-1} + \binom{n+2}{n}2^{n-2} + \ldots + \binom{2n}{n}2^n = 2^{2n}$$

State Mean: **0.17**

Board of Studies: Notes from the Marking Centre

(c)(i)
In general, the meaning of the word 'explain' was not understood and quite often responses did not have enough detail to demonstrate appropriate understanding of the problem.

Common problems were:

- While it was understood that that player A had to win 5 games out of 7, candidates could not explain the need for $\binom{6}{4}$ and not $\binom{7}{5}$
- incorrectly stating that A had to win the first game.

(ii)
Many candidates successfully applied the pattern from (c)(i) to this part to get the correct expression. Some candidates who did not provide a correct explanation in part (c)(i) were able to provide the correct expression in part (ii), demonstrating a weakness in being able to articulate mathematical logic.

From the phrase 'at most' many candidates incorrectly concluded that the answer must be 1 – [part (c)(i) answer] or some other similar expression.

(iii)
Most candidates struggled with this part, with only a small number of responses being awarded full marks.

Generally it was the candidates who were successful in part (c)(ii) who could generalise their answer and go on to prove the required result.

Many incorrect responses involved attempts to manipulate the Binomial theorem to force it into the form of the given expression. The requirement 'by considering the probability' seemed to have been ignored.