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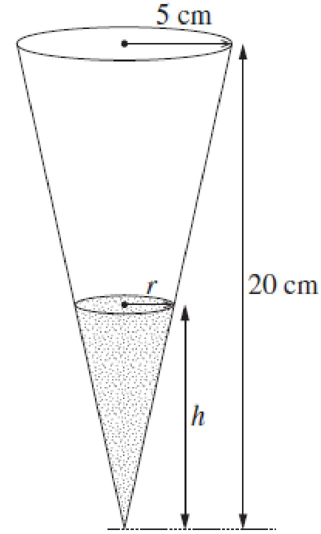
2016 **12 a** The diagram shows a conical soap dispenser of radius 5 cm and height 20 cm. At any time t seconds, the top surface of the soap in the container is a circle of radius r cm and its height is h cm.



The volume of the soap is given by $v = \frac{1}{3}\pi r^2 h$.

(i) Explain why $r = \frac{h}{4}$. **1**

(ii) Show that $\frac{dv}{dh} = \frac{\pi}{16}h^2$. **1**

The dispenser has a leak which causes soap to drip from the dispenser. The area of the circle formed by the top surface of the soap is decreasing at a constant rate of 0.04 cm^2s^{-1}.

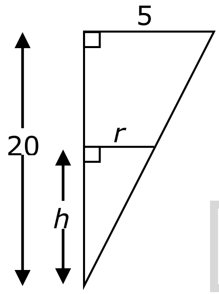
(iii) Show that $\frac{dh}{dt} = \frac{-0.32}{\pi h}$. **2**

(iv) What is the rate of change of the volume of the soap, with respect to time, when $h = 10$? **2**

(i) Using matching sides of similar triangles:

$$\frac{r}{5} = \frac{h}{20}$$

$$r = \frac{h}{4}$$



State Mean: **0.74**

(ii) $$v = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi\left(\frac{h}{4}\right)^2 h$$

$$\therefore v = \frac{\pi h^3}{48}$$

$$\frac{dv}{dh} = \frac{\pi}{16}h^2$$

State Mean: **0.76**

(iii) $$\frac{dA}{dt} = \frac{dA}{dh} \times \frac{dh}{dt}$$

$$A = \pi r^2 = \pi\left(\frac{h}{4}\right)^2 = \frac{\pi h^2}{16}$$

$$\frac{dA}{dh} = \frac{\pi}{8}h$$

$$-0.04 = \frac{\pi}{8}h \times \frac{dh}{dt}$$

$$\frac{dh}{dt} = -0.04 \div \frac{\pi}{8}h = \frac{-0.32}{\pi h}$$

State Mean: **1.13**

(iv) $$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$$

$$= \frac{\pi}{16}(10)^2 \times \frac{-0.32}{\pi(10)}$$

$$= -0.2$$

$\therefore$ the volume is decreasing at 0.2 cm^3s^{-1}.

State Mean: **1.42**

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by BOSTES.

BOSTES: Notes from the Marking Centre

)(i) In the large majority of responses, candidates wrote a proportion statement like $\frac{r}{5}=\frac{h}{20}$ and arrived at the result. Some candidates gave a full similarity proof which is more than is required for one mark.

Others made disconnected statements like $20\div 5=4 \therefore r=\frac{h}{4}$. Common problems were:

- not using mathematical concepts to justify the result required by the instruction 'explain' (or 'show')

(ii) In many responses, candidates could substitute h into the given equation to get $v=\frac{\pi h^3}{48}$ then differentiate with respect to h and arrive at the required result. Common problems were:

- differentiating and dealing with different variables, eg, differentiating (incorrectly) $v=\frac{1}{3}\pi r^2 h$ with respect to h to gain $\frac{dv}{dh}=\frac{1}{3}\pi r^2$, or with respect to r to gain $\frac{dv}{dh}=\frac{2}{3}\pi rh$, and then arriving at the required result
- unnecessarily using the quotient rule to differentiate $v=\frac{\pi h^3}{48}$

(iii) In a substantial number of responses, candidates deduced that $\frac{dA}{dh}=\frac{\pi h}{8}$ and found a relationship between $\frac{dh}{dt}$ and $\frac{dA}{dt}$ leading to an efficient two-line solution. Common problems were:

- using completely invalid logic
- poor understanding of the chain rule, often leading to 'fudging' results

(iv) Many responses included a valid proof using $\frac{dV}{dt}=\frac{dV}{dh}\times\frac{dh}{dt}$.

However, many responses appeared to have been worked starting with the given result $\frac{dh}{dt}=\frac{-0.32}{\pi h}$, eg,

$\frac{dh}{dt}=\frac{16}{\pi h^2}\times\frac{\square}{\square}=-\frac{0.32}{\pi h}$ so $\frac{\square}{\square}=-\frac{0.02h}{1}$, therefore $\frac{dV}{dt}=-0.02h$. Common problems were:

- using a variety of derivatives relating the variables in the question v, h, A (or S), r and t and putting them into chain rules hoping one would work
- making substitution and arithmetic errors