20 MX 1 **12 b**

When a particular biased coin is tossed, the probability of obtaining a head is $\frac{3}{5}$.

This coin is tossed 100 times.

Let X be the random variable representing the number of heads obtained. This random variable will have a binomial distribution.

(i) Find the expected value, $E(X)$. **1**

(ii) By finding the variance, Var(X), show that the standard deviation of X is approximately 5. **1**

(iii) By using a normal distribution, find the approximate probability that X is between 55 and 65. **1**

(i) $E(X) = \mu = np$

$= 100 \times \frac{3}{5}$

$= 60$ ✔

(ii) $Var(X) = np(1 - p)$

$= 60(1 - \frac{3}{5})$

$= 24$

$\sigma = \sqrt{24} \approx 5$ ✔

(iii) $z_1 = \frac{55 - 60}{5}$

$= -1$

$z_2 = \frac{65 - 60}{5}$

$= 1$

$55 \le P(X) \le 65 = -1 \le z \le 1$

≈ 0.68 ✔

State Mean:
0.95/1
0.88/1
0.71/1

HSC Marking Feedback

Part (b)(i)

Students should:

- understand the parameters associated with binomial distributions, ie $X \sim \text{Bin}(100, 0.6)$
- identify correct formula from Reference Sheet.

In better responses, students were able to:

- use the correct formula to arrive at the answer.

Areas for students to improve include:

- familiarising themselves with the Reference Sheet.

Part (b)(ii)

Students should:

- understand the relationship between variance and standard deviation
- calculate the variance and hence, the standard deviation.

In better responses, students were able to:

- substitute correctly in the formula for variance
- show that the standard deviation is $\sqrt{24}$.

Areas for students to improve include:

- focusing on using the Reference Sheet correctly.

Part (b)(iii)

Students should:

- link the probability to the standardised z-scores in normal distributions
- read-off the probability from the Reference Sheet.

In better responses, students were able to:

- use the empirical rule found in the Reference Sheet rather than attempting to calculate probabilities from z-scores.

Areas for students to improve include:

- reading the question carefully to ascertain when and how normal distribution approximation should be used.

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