20 MX 1 **14 a**

(i) Use the identity $(1 + x)^{2n} = (1 + x)^n(1 + x)^n$ to show that **2**

$\binom{2n}{n} = \binom{n}{0}^2 + \binom{n}{1}^2 + \ldots + \binom{n}{n}^2$ where n is a positive integer.

(ii) A club has $2n$ members, with n women and n men. **2**

A group consisting of an even number $(0, 2, 4, \ldots, 2n)$ of members is chosen, with the number of men equal to the number of women.

Show, giving reasons, that the number of ways to do this is $\binom{2n}{n}$.

(iii) From the group chosen in part (ii), one of the men and one of the women are selected as leaders. **2**

Show, giving reasons, that the number of ways to choose the even number of people and then the leaders is $1^2\binom{n}{1}^2 + 2^2\binom{n}{2}^2 + \ldots + n^2\binom{n}{n}^2$.

(iv) The process is now reversed so that the leaders, one man and one woman, are chosen first. The rest of the group is then selected, still made up of an equal number of women and men. **2**

By considering this reversed process, and using part (ii), find a simple expression for the sum in part (iii).

(i) $(1 + x)^{2n} = \binom{2n}{0} + \binom{2n}{1}x + \binom{2n}{2}x^2 + \ldots + \binom{2n}{n}x^n + \ldots$

$(1 + x)^n\,(1 + x)^n = [\binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \ldots + \binom{n}{n}x^n]\,[\binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \ldots + \binom{n}{n}x^n]$

Considering the coefficients of x^n in both expressions:

$$\binom{2n}{n}x^n = \binom{n}{0}\binom{n}{n}x^n + \binom{n}{1}x\binom{n}{n-1}x^{n-1} + \ldots + \binom{n}{n-1}x^{n-1}\binom{n}{1}x + \binom{n}{n}x^n\binom{n}{0}$$ ✓

As $\binom{n}{0} = \binom{n}{n}$, and $\binom{n}{1} = \binom{n}{n-1}$, etc, then $\binom{2n}{n} = \binom{n}{0}^2 + \binom{n}{1}^2 + \ldots + \binom{n}{n}^2$. ✓

(ii) Assume $2r$ members are chosen, which involves r women and r men.

There are $\binom{n}{r}$ ways of choosing the women and $\binom{n}{r}$ ways of choosing the men, which is $\binom{n}{r}^2$ ways. ✓

As r can take any value from 0 to n, then there are $\binom{n}{0}^2 + \binom{n}{1}^2 + \ldots + \binom{n}{n}^2 = \binom{2n}{n}$ ways. ✓

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(iii) In a group of r women there are r ways of choosing a leader, and similarly for men.

Hence, for a group of $2r$ people there are $\binom{n}{r}^2 \times r^2$, or $r^2\binom{n}{r}^2$. ✓

Again, as r can take any value from 0 to n, then there are $0^2\binom{n}{0}^2 + 1^2\binom{n}{1}^2 + \ldots + n^2\binom{n}{n}^2$ ways.

As $0^2\binom{n}{0}^2 = 0$, there are $1^2\binom{n}{1}^2 + \ldots + n^2\binom{n}{n}^2$ ways. ✓

(iv) If the leaders are chosen first then there are n ways of selecting a female leader, and n ways of selecting a male leader.

Also, there are now $2(n - 1)$ people left from which to choose $(n - 1)$ people. ✓

This means $n^2\binom{2(n-1)}{n-1}$ ways. ✓

State Mean:
0.52/2
0.46/2
0.52/2
0.31/2

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Part (a) (i)

Students should:

- recognise that the coefficients of terms in each of these expansions will be equal
- note that the term on the LHS of the required result is the coefficient of x^n in the expansion of $(1+x)^{2n}$.

In better responses, students were able to:

- clearly state that the coefficient of x^n in expansion of $(1+x)^n(1+x)^n$ is
$$\binom{n}{0}\binom{n}{n} + \binom{n}{1}\binom{n}{n-1} + \binom{n}{k}\binom{n}{n-k} + \cdots + \binom{n}{n}\binom{n}{0}$$
- explain why $\binom{n}{n} = \binom{n}{0}$ etc, using $\binom{n}{k} = \binom{n}{n-k}$.

Areas for students to improve include:

- demonstrating knowledge of the binomial expansions of these expressions.
- using the Pascal triangle relationship represented by $\binom{n}{k} = \binom{n}{n-k}$.

Part (a) (ii)

Students should:

- ensure that they fully justify how they are deriving the (supplied) answer.

In better responses, students were able to:

- explain, using a pattern, how each case could be generated
- add these cases together to derive the required result.

Areas for students to improve include:

- reading the question carefully to gain a full understanding of the situation
- taking time to build the answer step by step
- knowing when cases should be multiplied and added.

Part (a) (iii)

Students should:

- ensure that they fully justify how they are deriving the (supplied) answer.

In better responses, students were able to:

- explain, using a pattern, how each case could be generated
- clearly differentiate between the leader and the group formations
- add these cases together to derive the required result.

Areas for students to improve include:

- reading the question carefully to gain a full understanding of the situation
- taking time to build the answer step by step
- knowing when cases should be multiplied and added.

Part (a) (iv)

Students should:

- note that the choices for the leadership group will be the same in all cases
- recognise that the number of men and women from which to choose the groups has now been reduced by one
- recognise that the reversed method will, ultimately, generate an equivalent number of choices as the method in part (iii).

In better responses, students were able to:

- express the choices for leaders as $\binom{n}{1}^2 = n^2$
- express the choices for the members of the groups in the form $\binom{n-1}{k}^2$
- multiply these parts to generate an expression for each individual case
- add these cases together to derive a correct result

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- use the result from part (i) to simplify this summation.

Areas for students to improve include:

- reading the question carefully to gain a full understanding of the situation
- taking time to build the answer step by step
- knowing when cases should be multiplied and added
- realising that parts of questions are often connected, finding these connections and using them.

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