MX SP **11 b** A function $f(x)$ is given by $x^2 + 4x + 7$.

(i) Explain why the domain of the function $f(x)$ must be restricted if $f(x)$ is to have an inverse function. **1**

(ii) Give the equation for $f^{-1}(x)$ if the domain of $f(x)$ is restricted to $x \geq -2$. **2**

(iii) State the domain and range of $f^{-1}(x)$, given the restriction in part (b). **2**

(iv) Sketch the curve $y = f^{-1}(x)$. **2**

(i) $f(x) = x^2 + 4x + 7$ is a parabola.

Except for the minimum value, all other $f(x)$ values in the range has two x-values. This means the horizontal line test fails and so the domain needs to be restricted.

(ii) Let $y = x^2 + 4x + 7$.

$\therefore$ for $f^{-1}(x)$: $x = y^2 + 4y + 7$.

$$y^2 + 4y = x - 7$$

$$y^2 + 4y + 4 = x - 7 + 4$$

$$(y + 2)^2 = x - 3$$

$$y + 2 = \pm\sqrt{x-3}$$

$$y = -2 \pm \sqrt{x-3}$$

But, as domain for $f(x)$ is $x \geq -2$, then range of $f^{-1}(x)$ is $y \geq -2$.

$$\therefore\ y = -2 + \sqrt{x-3}$$

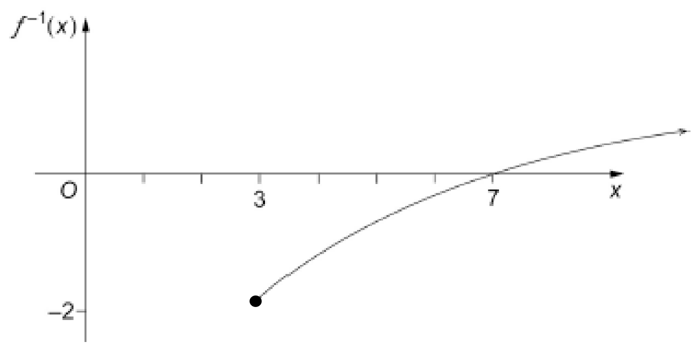
$$\therefore\ f^{-1}(x) = -2 + \sqrt{x-3}$$

(iii) For $f^{-1}(x)$:

Domain: $x \geq 3$, or $[3, \infty)$

Range: $y \geq -2$, or $[-2, \infty)$

(iv)



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