**MX SP 12 b** A force described by the vector $\underset{\sim}{F} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ newtons is applied to an object lying on a line $\ell$ which is parallel to the vector $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

(i) Find the component of the force $\underset{\sim}{F}$ in the direction of the line $\ell$. **2**

(ii) What is the component of the force $\underset{\sim}{F}$ in the direction perpendicular to the line? **1**

(i) Let $\underset{\sim}{v} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$\therefore$ unit vector parallel to $\underset{\sim}{v}$ is $\hat{\underset{\sim}{v}}$.

$$\hat{\underset{\sim}{v}} = \frac{\underset{\sim}{v}}{\left|\underset{\sim}{v}\right|} = \frac{\begin{pmatrix} 3 \\ 4 \end{pmatrix}}{\left|\begin{pmatrix} 3 \\ 4 \end{pmatrix}\right|}$$

$$= \frac{\begin{pmatrix} 3 \\ 4 \end{pmatrix}}{\sqrt{3^2 + 4^2}}$$

$$= \frac{1}{5}\begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 0.6 \\ 0.8 \end{pmatrix}$$

For the projection of $\underset{\sim}{F}$ on $\underset{\sim}{v}$:

$$proj_{\hat{\underset{\sim}{v}}}\underset{\sim}{F} = \frac{\hat{\underset{\sim}{v}} \cdot \underset{\sim}{F}}{\left|\hat{\underset{\sim}{v}}\right|^2} \cdot \hat{\underset{\sim}{v}}$$

Now, $\hat{\underset{\sim}{v}} \cdot \underset{\sim}{F} = \begin{pmatrix} 0.6 \\ 0.8 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$= 0.6 \times 2 + 0.8 \times 1$

$= 2$

Also, $\left|\hat{\underset{\sim}{v}}\right|^2 = \sqrt{0.6^2 + 0.8^2}$

$= 1$

$\therefore \; proj_{\hat{\underset{\sim}{v}}}\underset{\sim}{F} = \frac{2}{1} \cdot \hat{\underset{\sim}{v}}$

$= 2\begin{pmatrix} 0.6 \\ 0.8 \end{pmatrix}$

$= \begin{pmatrix} 1.2 \\ 1.6 \end{pmatrix}$

(ii) As $\underset{\sim}{F} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, then

$\begin{pmatrix} 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 1.2 \\ 1.6 \end{pmatrix} = \begin{pmatrix} 0.8 \\ -0.6 \end{pmatrix}$.

[Checking, $\begin{pmatrix} 1.2 \\ 1.6 \end{pmatrix} . \begin{pmatrix} 0.8 \\ -0.6 \end{pmatrix} = 0$]