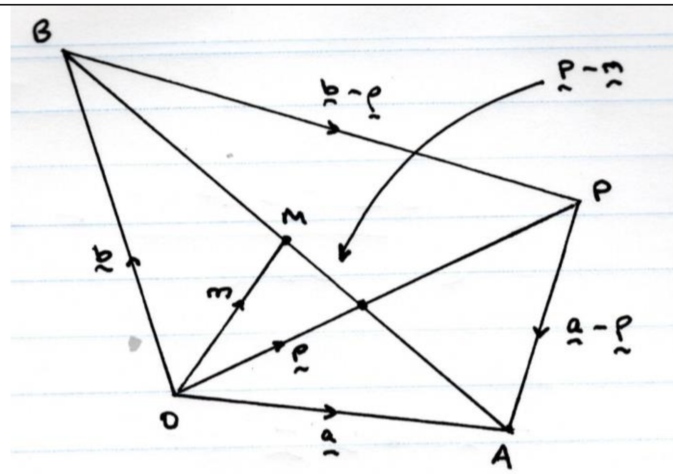
MX SP	**12 c**	The points A and B are fixed points in a plane and have position vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ respectively. The point P with position vector $\underset{\sim}{p}$ also lies in the plane and is chosen so that $\angle APB = 90^\circ$.	
		(i) Explain why $(\underset{\sim}{a} - \underset{\sim}{p})\cdot(\underset{\sim}{b} - \underset{\sim}{p}) = 0$	**1**
		(ii) Let $\underset{\sim}{m} = \frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{b})$ denote the position vector of M, the midpoint of A and B. Using the properties of vectors, show that $\lvert\underset{\sim}{p} - \underset{\sim}{m}\rvert^2$ is independent of $\underset{\sim}{p}$ and find its value.	**3**
		(iii) What does the result in part (ii) prove about the point P?	**1**



(i) From the diagram,

$\overrightarrow{PA} = \underset{\sim}{a} - \underset{\sim}{p}$ and $\overrightarrow{PB} = \underset{\sim}{b} - \underset{\sim}{p}$.

As $\overrightarrow{PA} \perp \overrightarrow{PB}$, then $(\underset{\sim}{a} - \underset{\sim}{p})\cdot(\underset{\sim}{b} - \underset{\sim}{p}) = 0$.

(ii) As $(\underset{\sim}{a} - \underset{\sim}{p})\cdot(\underset{\sim}{b} - \underset{\sim}{p}) = 0$, then

$\underset{\sim}{a}\cdot\underset{\sim}{b} - \underset{\sim}{p}\cdot(\underset{\sim}{a} + \underset{\sim}{b}) + \underset{\sim}{p}\cdot\underset{\sim}{p} = 0$ …. *

Now, $\lvert\underset{\sim}{p} - \underset{\sim}{m}\rvert^2 = \lvert\underset{\sim}{p} - \frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{b})\rvert^2$

$= \lvert\frac{1}{2}(2\underset{\sim}{p} - (\underset{\sim}{a} + \underset{\sim}{b})\rvert^2$

$= \frac{1}{4}\lvert 2\underset{\sim}{p} - (\underset{\sim}{a} + \underset{\sim}{b})\rvert^2$

$= \frac{1}{4}[(2\underset{\sim}{p} - (\underset{\sim}{a} + \underset{\sim}{b})\cdot(2\underset{\sim}{p} - (\underset{\sim}{a} + \underset{\sim}{b}))]$

$= \frac{1}{4} \times 4\underset{\sim}{p}\cdot\underset{\sim}{p} - \frac{1}{4} \times 4\underset{\sim}{p}(\underset{\sim}{a} + \underset{\sim}{b}) + \frac{1}{4} \times (\underset{\sim}{a} + \underset{\sim}{b})\cdot(\underset{\sim}{a} + \underset{\sim}{b})$

$= \underset{\sim}{p}\cdot\underset{\sim}{p} - \underset{\sim}{p}(\underset{\sim}{a} + \underset{\sim}{b}) + \frac{1}{4}(\underset{\sim}{a} + \underset{\sim}{b})\cdot(\underset{\sim}{a} + \underset{\sim}{b})$

But, from *, $\underset{\sim}{p}\cdot(\underset{\sim}{a} + \underset{\sim}{b}) = \underset{\sim}{a}\cdot\underset{\sim}{b} + \underset{\sim}{p}\cdot\underset{\sim}{p}$,

hence, $\lvert\underset{\sim}{p} - \underset{\sim}{m}\rvert^2 = \underset{\sim}{p}\cdot\underset{\sim}{p} - [\underset{\sim}{a}\cdot\underset{\sim}{b} + \underset{\sim}{p}\cdot\underset{\sim}{p}] + \frac{1}{4}(\underset{\sim}{a} + \underset{\sim}{b})\cdot(\underset{\sim}{a} + \underset{\sim}{b})$

$= \frac{1}{4}(\underset{\sim}{a} + \underset{\sim}{b})\cdot(\underset{\sim}{a} + \underset{\sim}{b}) - \underset{\sim}{a}\cdot\underset{\sim}{b}$, which is independent of $\underset{\sim}{p}$.

(iii) As $\lvert\underset{\sim}{p} - \underset{\sim}{m}\rvert^2 = \frac{1}{4}(\underset{\sim}{a} + \underset{\sim}{b})\cdot(\underset{\sim}{a} + \underset{\sim}{b}) - \underset{\sim}{a}\cdot\underset{\sim}{b}$, then P lies on a circle, centre at origin with radius $\sqrt{\frac{1}{4}(\underset{\sim}{a}+\underset{\sim}{b})\cdot(\underset{\sim}{a}+\underset{\sim}{b}) - \underset{\sim}{a}\cdot\underset{\sim}{b}}$ units. Other solutions at

https://educationstandards.nsw.edu.au/wps/wcm/connect/0ca974bc-60ce-4be4-9410-

6e2351ca8049/mathematics-ext-1-sample-examination-materials-2020.pdf?MOD=AJPERES&CVID=

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by NESA.