



MX 12 Use mathematical induction to prove that $2^{3n} - 3^n$ is divisible by 5 for $n \geq 1$. **3**
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Test $n = 1$: $2^3 - 3^1 = 8 - 3$
 $= 5$, which is divisible by 5 \therefore true for $n = 1$

Let $n = k$ be a value for which the result is true.

Let $2^{3k} - 3^k = 5M$, where M is an integer.

Now prove true for $n = k + 1$

$$\begin{aligned} 2^{3(k+1)} - 3^{k+1} &= 2^{3k+3} - 3^{k+1} \\ &= 2^{3k} \cdot 2^3 - 3^k \cdot 3 \\ &= 8 \times 2^{3k} - 3 \cdot 3^k \\ &= 5 \times 2^{3k} + 3 \times 2^{3k} - 3 \cdot 3^k \\ &= 5 \times 2^{3k} + 3(2^{3k} - 3^k) \\ &= 5 \times 2^{3k} + 3(5M) \\ &= 5[2^{3k} + 3M], \text{ which is divisible by 5} \quad \therefore \text{ true for } n = k + 1 \end{aligned}$$

\therefore by the principle of mathematical induction, the result is true for all integers $n \geq 1$.

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