

MX 13
SP a

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Use the substitution $x = \sin^2 \theta$ to determine $\int_0^{\frac{1}{2}} \sqrt{\frac{x}{1-x}} dx$.

$$\int_0^{\frac{1}{2}} \sqrt{\frac{x}{1-x}} dx = \int_0^{\frac{\pi}{4}} \sqrt{\frac{\sin^2 \theta}{1-\sin^2 \theta}} \cdot 2 \sin \theta \cos \theta d\theta$$

$$= \int_0^{\frac{\pi}{4}} \sqrt{\frac{\sin^2 \theta}{\cos^2 \theta}} \cdot 2 \sin \theta \cos \theta d\theta$$

$$= 2 \int_0^{\frac{\pi}{4}} \frac{\sin \theta}{\cos \theta} \cdot \sin \theta \cos \theta d\theta$$

$$= 2 \int_0^{\frac{\pi}{4}} \sin^2 \theta d\theta$$

$$= 2 \int_0^{\frac{\pi}{4}} \frac{1}{2} (1 - \cos 2\theta) d\theta$$

$$= \int_0^{\frac{\pi}{4}} (1 - \cos 2\theta) d\theta$$

$$= \left[\theta - \frac{1}{2} \sin 2\theta \right]_0^{\frac{\pi}{4}}$$

$$= \frac{\pi}{4} - \frac{1}{2} \sin 2\left(\frac{\pi}{4}\right) - 0$$

$$= \frac{\pi}{4} - \frac{1}{2}$$

$$x = \sin^2 \theta$$

$$\frac{dx}{d\theta} = 2 \sin \theta \cos \theta$$

$$dx = 2 \sin \theta \cos \theta d\theta$$

$$\text{Also, } x = \frac{1}{2}, \theta = \frac{\pi}{4}; x = 0, \theta = 0$$

* These solutions have been provided by [projectmaths](http://projectmaths.com.au) and are not supplied or endorsed by NESA.

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