MX 13 a **SP** 3

Use the substitution $x = \sin^2\theta$ to determine $\int_0^{\frac{1}{2}} \sqrt{\frac{x}{1-x}}\,dx$.

$$\int_0^{\frac{1}{2}} \sqrt{\frac{x}{1-x}}\,dx = \int_0^{\frac{\pi}{4}} \sqrt{\frac{\sin^2\theta}{1-\sin^2\theta}}\,.2\sin\theta\cos\theta\;d\theta$$

$$= \int_0^{\frac{\pi}{4}} \sqrt{\frac{\sin^2\theta}{\cos^2\theta}}\,.2\sin\theta\cos\theta\;d\theta$$

$$= 2\int_0^{\frac{\pi}{4}} \frac{\sin\theta}{\cos\theta}\,.\sin\theta\cos\theta\;d\theta$$

$$= 2\int_0^{\frac{\pi}{4}} \sin^2\theta\,.d\theta$$

$$= 2\int_0^{\frac{\pi}{4}} \frac{1}{2}(1-\cos 2\theta)\,.d\theta$$

$$= \int_0^{\frac{\pi}{4}} \left(1-\cos 2\theta\right).d\theta$$

$$= \left[\theta - \frac{1}{2}\sin 2\theta\right]_0^{\frac{\pi}{4}}$$

$$= \frac{\pi}{4} - \frac{1}{2}\sin 2(\frac{\pi}{4}) - 0$$

$$= \frac{\pi}{4} - \frac{1}{2}$$

$x = \sin^2\theta$

$\frac{dx}{d\theta} = 2\sin\theta\cos\theta$

$dx = 2\sin\theta\cos\theta\;d\theta$

Also, $x = \frac{1}{2}$, $\theta = \frac{\pi}{4}$; $x = 0$, $\theta = 0$

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by NESA.